# Phase diagram and dimensional reduction in 5d Yang-Mills

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# What if there really is a 5th dimension??

- Lots of Extra-dimensional phenomenology available.
  - Mostly perturbative/stringy.
  - What non-perturbative do we know about extra-dimensional YM?
- What does non-perturbative extra-dimensional YM even mean??
  - Non-renormalizable $\longrightarrow$ no continuum limit!
    - Observables depend on the details of the UV-completion.
  - Effective theory ($L_5^{-1}$) of a more fundamental one ($\Lambda_f$):
    - If scale difference $L_5 \gg 1/\Lambda_f$, dependence on the UV-completion suppressed.
- Might make sense if

$$L_5 \gg 1/\Lambda_f.$$

Related work:Arkani-Hamed et al. hep-ph/0104005, Farakos et al. hep-lat/0207242, Ejiri et al.hep-lat/0204022, Irges & Knechtli 0905.2757, Beard et al. hep-lat/9709120, Poster by Petrov... Here, emphasis on dimensional reduction.

# The Model

- To make it simple: SU(2) on $\mathbb{R}^4 \times \mathrm{S}^1$ on a $(= N_s^4 \times N_5)$ Lattice:

$$S_E^L = \frac{\beta_5}{\gamma} \sum_{1 \le M < N \le 4} \left[ 1 - \frac{1}{2} \mathrm{ReTr}\, P_{MN}(x) \right] + \gamma \beta_5 \sum_{M=1}^{4} \left[ 1 - \frac{1}{2} \mathrm{ReTr}\, P_{M5}(x) \right]$$

- Strategy: Take the continuum limit in the 5th direction $N_5 \to \infty, N_5/\gamma = \text{fixed}$.

  Note: Opposite to exploration of layered phase by Farakos et al.

  Pro:
  - Continuum theory defined by 2 dim.less ratios $(g_5^2 \Lambda_f,\, L_5 \Lambda_f)$ $\longrightarrow$ Lattice theory defined by 2 parameters:

$$\beta_5 = \frac{4a}{g_5^2}, \qquad \widetilde{N}_5 \equiv N_5/\gamma \sim L_5/a + \dots$$

  - Remove discretization error from 5th direction.

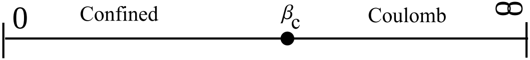
  Con: Lose 5d hypercubic symmetry of action $\to L_5/a = ?$

# Outline

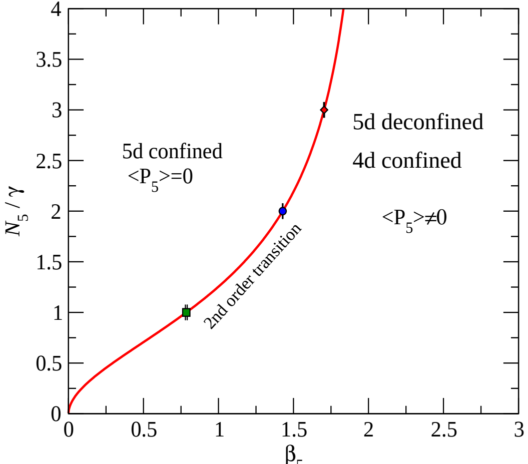
- Phase diagram
- Dimensional reduction
- Lines of Constant (4d) Physics
- Conclusions

# Phase diagram

Isotropic non-compactified:




- Expect deconfinement when corr. length $\sim$ compact. radius.
- In Coulomb phase: Infinite correlation length
  $\longrightarrow$ Deconfines at *all* compactification radii.



# Dimensional reduction in the deconfined phase

Zero modes at large distances ($\Delta x \gg L_5$) described by a 4d continuum theory:

$$S_{\rm eff} = \frac{1}{g_4^2(L_5)} \int \mathrm{d}^4x \left[\frac{1}{2}\mathrm{Tr}F_{\mu\nu}^2 + \mathrm{Tr}[D_\mu A_5]^2 + m_5^2 \mathrm{Tr}A_5^2 + \lambda \mathrm{Tr}\, A_5^4 \ldots\right]$$

This theory is renormalizable continuum theory $\rightarrow$ We know something about it:

- Confinement:

$$\sigma \neq 0$$

- Asymptotic freedom:

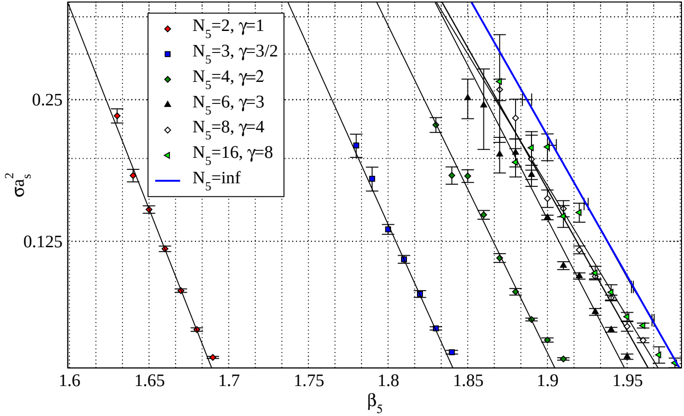
$$\sigma \sim \frac{1}{{L_5}^2} \exp\left[-\frac{1}{b_0}\frac{1}{g_4^2(L_5)}\right]$$

For a start, match in PT ...

$$g_{4d}^2(L_5) \sim \frac{g_5^2}{L_5} = \frac{4}{\beta_5 \widetilde{N}_5}$$

. . . to get

$$\sigma a^2 \sim \frac{a^2}{L_5^2} \exp\left[ -\frac{1}{2b_0}\frac{1}{g_4^2(L_5)} \right] \sim \frac{1}{\widetilde{N}_5^2} \exp\left[ -\frac{1}{4b_0}\beta_5 \times \widetilde{N_5} \right] \qquad (1)$$




$$\widetilde{N}_5 = N_5/\gamma = 2, \quad V = 6^4 \times N_5$$

What does this mean??

# Inverse dimensional reduction!

- Increasing the $L_5/a$ linearly makes the 4d correlation length $\xi_{4d} = \sigma^{-1/2}$ grow exponentially:

$$L_5/a = \widetilde{N}_5 \qquad \xi_{4d}/a \sim \widetilde{N}_5 \exp(\frac{1}{2b_0}\widetilde{N}_5\beta) \tag{2}$$

$\longrightarrow$ Dimensional reduction to 4d at **LARGE** $\widetilde{N}_5 = L_5/a$!!

Turn the argument around: 4d physics fixed

- Increasing scale separation $L_5/a$ makes the 5th dimension exponentially small in 4d units

$$\xi_{4d} = \text{fixed} \qquad L_5/\xi_{4d} \sim \exp(-\frac{1}{2b_0}\widetilde{N}_5\beta) \tag{3}$$

If you know $\beta$ and $a/\xi$, the extent of 5th dim. is fixed. $(a_5/\xi_{4d} = M_P^{-1}/\text{GeV}^{-1}, \beta > 1.65 \rightarrow L_5 \sim 10M_P^{-1})$

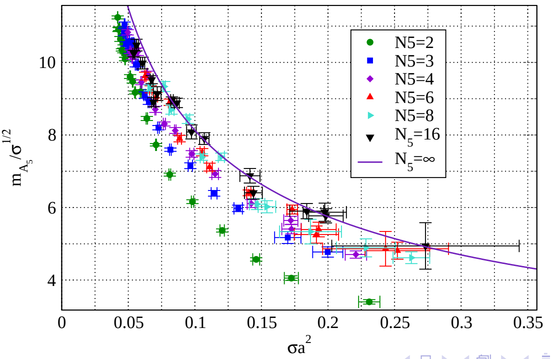
# What happened to $A_5$?

...or how to predict the physical New Physics scale if one sees adjoint scalars with mass $m_5$ with self-coupling $\lambda$??

- Theory defined by two parameters: measure $\lambda$ and $m_5/\sqrt{\sigma}$ $\longrightarrow$ fully predictive.

To leading order in PT:

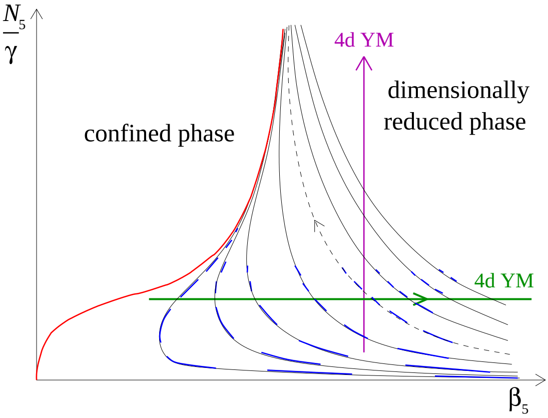
$$m_5 a \sim \sqrt{\frac{g_5^2}{L_5}\frac{a}{L_5}} \sim \sqrt{\frac{1}{\beta_5 \widetilde{N}_5}\frac{1}{\widetilde{N}_5}}, \qquad \lambda \sim \frac{g_5^4}{L_5^2} \sim \frac{1}{(\beta \widetilde{N}_5)^2} \tag{4}$$



# How about continuum limit?

What happens if we try to take take the continuum limit keeping the low energy sector $= m_5/\sqrt{\sigma}$ fixed?

$$m_5^{-1}\sqrt{\sigma} \sim \sqrt{\beta_5 \widetilde{N}_5} \exp\left[-\frac{1}{2b_0}\widetilde{N}_5\beta_5\right] \tag{5}$$



## Conclusions

- Dimensional reduction can be used to understand the deconfined phase of 5dYM on a lattice.
- The theory makes sense as a low energy model of high dimensional theories if $L_5 \gg \Lambda_f^{-1}$.
- and can be used to make non-perturbative prediciton for onset scale for new physics.

$$L_5 \gg \Lambda_f^{-1} \longrightarrow \xi_{4d} \gg L_5 \tag{6}$$

- Various continuum limits available, none of them 5d.

## Outlook

- Higher rank groups, Hosotani mechanism
- Boundary conditions, Orbifold
- Fermions