Cold Quark Matter

or Neutron stars to 3-loops

Aleksi Kurkela
with Paul Romatschke and Aleksi Vuorinen
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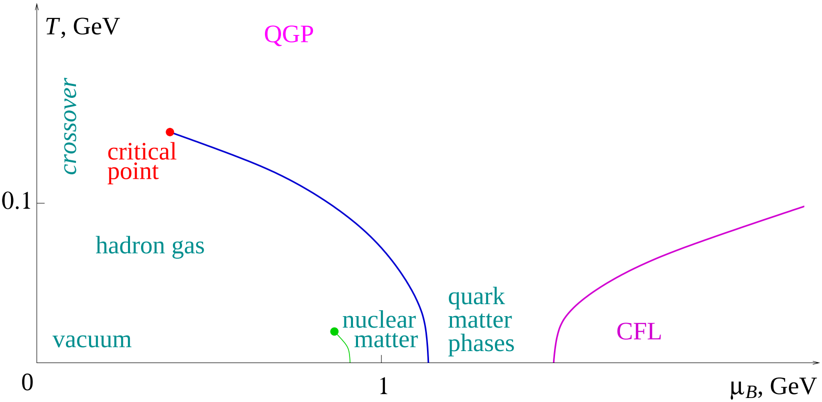
Outline

- Introduction
- Computation of the thermodynamics of QCD to 3-loops with m_s.
- Thermodynamics of charge neutral quark matter in beta equilibrium
- Phase transition between hadronic and quark matter
- Strange quark matter hypothesis
- Neutron, Quark and Hybrid stars

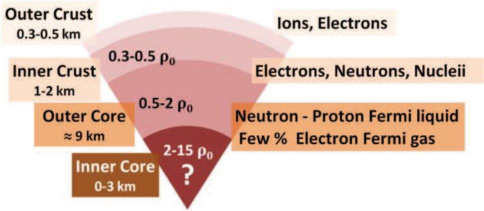
Introduction

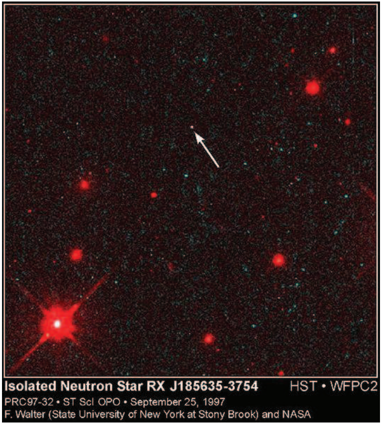
Ultimate goal to understand the whole phase diagram of strongly interacting matter:



Compact stars

- Masses $\lesssim 2.0 M_\odot$
- Radii $\sim$ 15km
- $T <$ KeV
- $n \lesssim 15\rho_0 \quad (\rho_0 = 0.16\mathrm{fm}^{-3})$





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  - At low densities ($n_B \lesssim 0.16\mathrm{fm}^{-3}, \mu_B = 3\mu_q \sim 1\mathrm{GeV}$): Quantum many-body theory, Dynamics of nucleons, hyperons, etc.
  - At (asymptotically) high densities $\alpha_s(\mu) \sim 1/\log(\mu^2)$ $\longrightarrow$ Perturbation theory.

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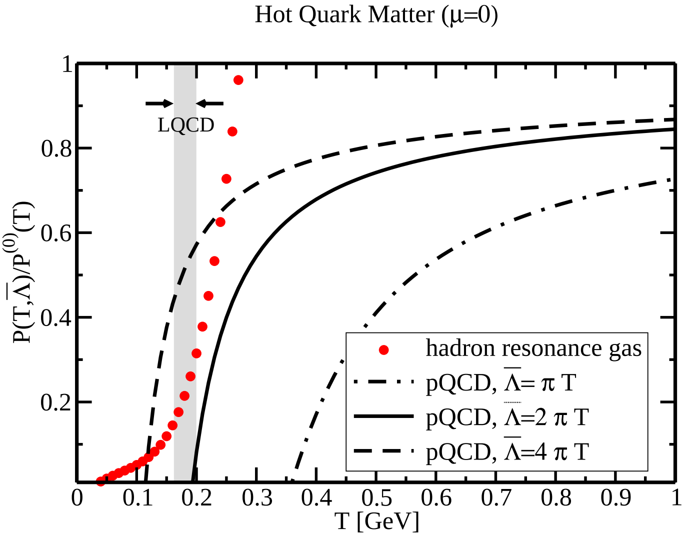
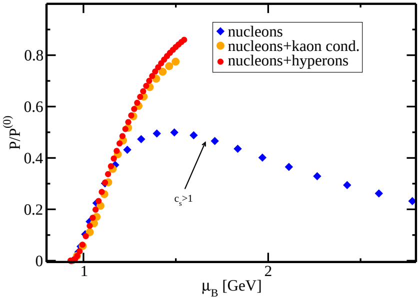
- Stategy: Interpolate between low and high densities to obtain a unified description
  - Relies on not having an exotic phase in between (Skyrme cristal, quarkyonic matter. . .)

Inspiration from finite T-case



At low densities ($n_B \lesssim 0.16\mathrm{fm}^{-3}, \mu_B = 3\mu_q \sim 1\mathrm{GeV}$):

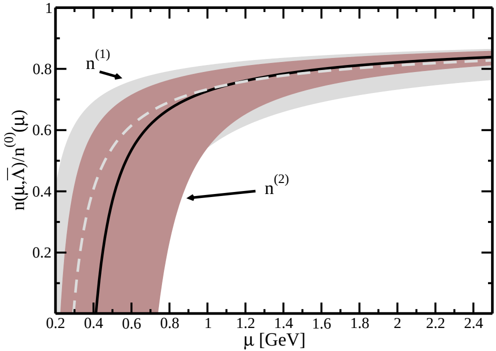


- Write down 2- and 3-body hamiltonian for nuclei
- Match them to experimentally known potentials:
    - 2-body: Argonne v_{18}
    - 3-body: Urbana IX. . .
    - Properties of nuclei and hypernuclei . . .
- Solve many body Schrödinger equation for the EoS
- Hope for the best, fear for the worst. . .

At high densities, $\alpha_s(\mu) \sim 1/\log(\mu^2)$

Weak coupling expansion of QCD pressure with $m_{\text{quark}} = 0$ known to $\mathcal{O}({\alpha_s}^2)$ (Freedman, McLerran)

(a)



- 2-loop computation shows that m_s-effects large at low/intermediate densities
  - This work: 3-loops with $m_s \neq 0$

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Perturbative evaluation of the grand potential

Thermodynamics defined by the grand potential:

$$\begin{aligned}\Omega(\mu_u,\mu_d,\mu_s,m_s) &= 1/V_4 \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{QCD}} \\ \mathcal{L}_{QCD} &= \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i .\end{aligned}$$

All the thermodynamical quantities can be derived from $\Omega(\mu_u,\mu_d,\mu_s,m_s)$:

$$\begin{aligned} pV &= -\Omega(\mu_u,\mu_d,\mu_s,m_s) \\ n_i &= -\partial_{\mu_i}\Omega(\mu_u,\mu_d,\mu_s,m_s) \\ \varepsilon &= -p + n_i\mu_i \end{aligned}$$

Perturbative evaluation of the grand potential

Thermodynamics defined by the grand potential:

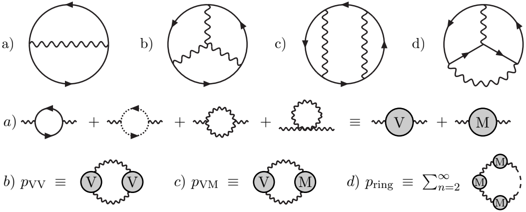
$$\Omega(\mu_u, \mu_d, \mu_s, m_s) \quad = \quad 1/V_4 \log \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu e^{-\int d^4x \mathcal{L}_{QCD}}$$

$$\mathcal{L}_{QCD} \quad = \quad \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}_i(\gamma_\mu D_\mu + m_i - \mu_i\gamma_0)\psi_i .$$

$$= \quad \frac{1}{i\not{P} + m} \longrightarrow \frac{1}{i\left[\gamma_0(p_0 + i\mu) + \vec{\gamma}\cdot\vec{p}\right] + m}$$

$$= \quad \frac{\delta_{\mu\nu}}{P^2} ,$$

- To get Ω to α_s^2:1PI vacuum bubbles:

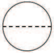


After scalarization end up with integrals constructed from μ-dep. massive scalar propagators:

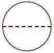
$$\Delta(p) = \frac{1}{(p_0 + i\mu)^2 + \vec{p}^2 + m_s^2}$$

- Technical challenge, Multiscale problem!:
  - $\mu = 0$: Extensive literature on integral reduction, IBP.
    - Ready-to-use applications FIRE, Tarcer..
  - $m = 0$: 3d Fourier transformation technology.
  - Combination of these problematic. Here, the stategy: reduce the μ-dependent vacuum bubbles to $\mu = 0$ amplitudes with funny μ-dependent "kinematic" integrals.
    - Reduce the amplitudes using FIRE and take the masters from literature.

Example:

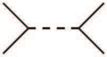


$\rightarrow$



$$-\,2\int \frac{d^3\vec{p}}{(2\pi)^3}\,\frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}$$

$$+\int \frac{d^3\vec{p}}{(2\pi)^3}\,\frac{\theta(\mu - E(\vec{p}))}{2E(\vec{p})}\int \frac{d^3\vec{q}}{(2\pi)^3}\,\frac{\theta(\mu - E(\vec{q}))}{2E(\vec{q})}$$



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"Bag constant" and the CSC mass gap:

- The any computation gives the pressure up to an additional constant:

$$P(\mu,\bar{\Lambda}) = -B + \int_{\mu_0(\bar{\Lambda})}^{\mu} \mathrm{d}\mu \; n(\mu,\bar{\Lambda})$$

  - Define $P(\mu = 0) = 0$. Not possible for a perturbative computation...
- In addition, if the theory is in a gapped CSC phase, an extra term appears in the pressure:

$$P_{CSC} \quad \leq \quad \frac{\Delta^2 \left(\mu_u + \mu_d + \mu_s\right)^2}{3\pi^2} \, ,$$

  - Asymptotically $\Delta \sim \frac{\mu}{g^5} \, \mathrm{e}^{-3\pi^2/(2\sqrt{g})}$,
  - Check the effect by varying $\Delta = 0 \ldots 100\mathrm{MeV}$.

Thermodynamics of charge neutral quark matter in beta-equilibrium

In any physically realistic situation, the quark matter is

- in beta-equilibrium due to the weak interactions

$$\begin{aligned} d \to u + e + \bar{\nu}_e, &\qquad u + e \to d + \nu_e, \\ s \to u + e + \bar{\nu}_e, &\qquad u + e \to s + \nu_e, \\ s + u \quad \leftrightarrow &\quad d + u, \end{aligned}$$

  leading to a constraint for the quark chemical potentials

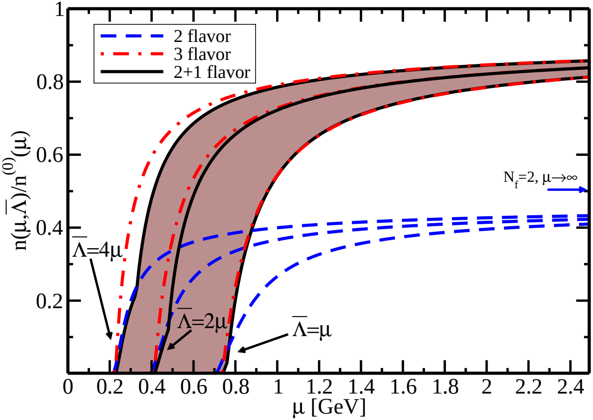
  (neglecting the neutrinos)

$$\mu \equiv \mu_s = \mu_d = \mu_u + \mu_e.$$

- and charge neutral:

$$2/3 n_u - 1/3 n_s - 1/3 n_d - n_e = 0$$

Thermodynamics of charge neutral quark matter in beta-equilibrium



- at μ_c all quark number densities go to zero.
- at μ_s strange quark number density goes to zero.
- $\mu_s < \mu < \infty$: Interpolates smoothly between $N_f = 2$ and $N_f = 3$.

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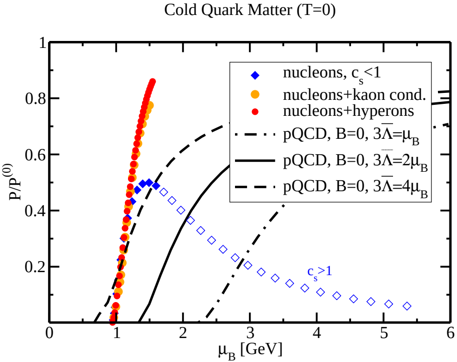
If there are no exotic phases, there will be a phase transition between the hadronic and quark matter phases at some μ_{pt}.

- Catalog all possible EoS:s $(B, \bar{\Lambda})$ fulfilling:
  - Equal pressure for both phases at the phase transition
  - Monotonically increasing energy density



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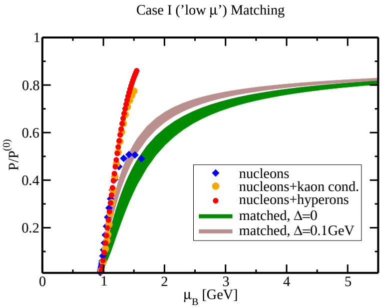
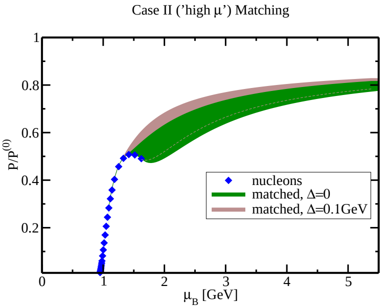
- Matching possible in two disjoint regimes:


Case I ('low μ') Matching

$$0.16\mathrm{fm}^{-3} < n_B \lesssim 0.32\mathrm{fm}^{-3}$$


Case II ('high μ') Matching

$$n_B > 0.64\mathrm{fm}^{-3}$$

⟶ Represents the best educated guess available for the true EoS

- Matching reduces significantly the perturbative uncertainties

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Strange quark matter hypothesis (or $\mu_{pt} < 0.31\text{GeV}$)

If the energy per baryon in quark matter is less than

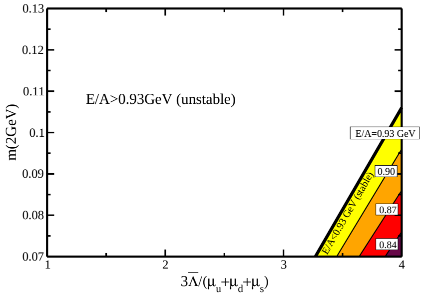
$$E/A = 3\mu_c = 0.93\text{GeV} \qquad {}^{56}\text{Fe}$$

then quark matter is the true ground state → Nuclear matter metastable.

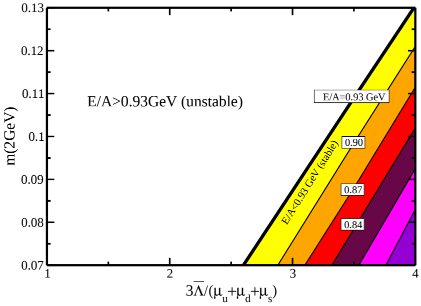
- Lifetime:
  - Nucleons → 2 Flavor quark matter:
    - Equilibration through Strong interaction $\longrightarrow t_{relax} \sim 1/\Lambda_{QCD}$
    - Short-lived. Ruled out by experiment!
  - Nucleons → 3 Flavor quark matter:
    - Equilibration through Weak Interactions (10^{60} years for $A > 6$)
    - Adding d.o.fs increases pressure → more likely to be stable
    - Experimentally plausible, let's find out what the theory says!

Strategy: Find out if SQM stable in the parameter space $(m_s, B, \bar{\Lambda})$ with

- $n_s > 0$ (quark mass extremely important!)
- $\mu_c < 0.31\text{GeV}$



Normal Quark matter, Δ=0, $\Lambda_{\overline{MS}}$=0.378GeV



CSC, Δ=100 MeV, $\Lambda_{\overline{MS}}$=0.378GeV

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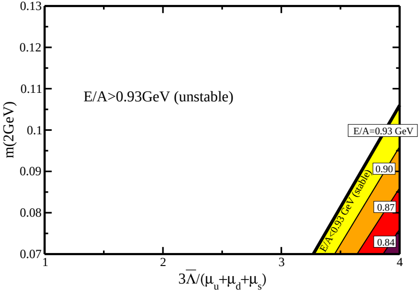

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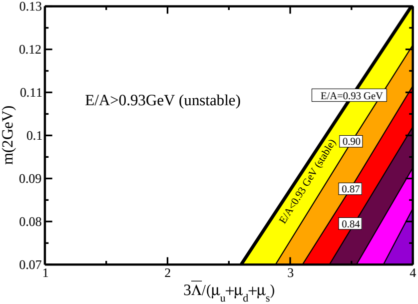

- Parameter space very hostile for $(\Delta = 0)$. Including CSC makes SQM more plausible
  $\longrightarrow$ Stable SQM disfavored but not ruled out.

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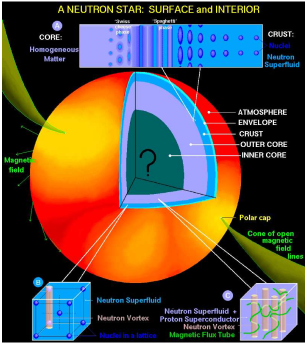
Compact stars

- The mass-radius relationship is very sensitive to the EoS
  - M-R relation given by TOV-equation:

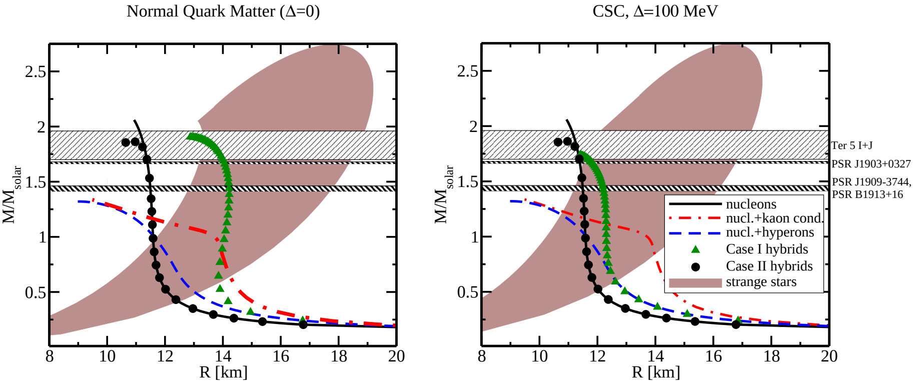
$$
\begin{aligned}
dM(r) &= 4\pi r^2 \varepsilon(r) dr, \\
dP(r) &= -\frac{G\left(P(r)+\varepsilon(r)\right)\left(M(r)+4\pi r^3 P(r)\right)}{r\left(r-2GM(r)\right)} dr \,,
\end{aligned}
$$

  - Takes $\varepsilon(p)$ as an input.

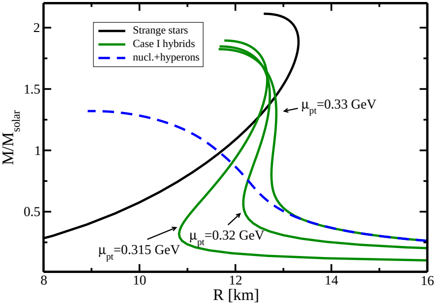


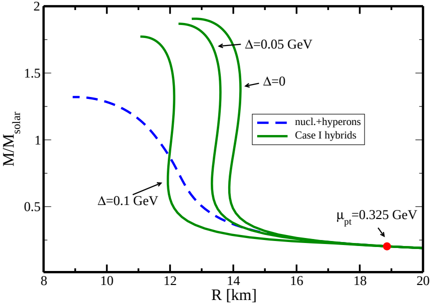
Let's consider compact stars:

- Made of nucleonic matter
- Made of pure quark matter
- Hybrid stars with
  - Mainly quark matter with thin nucleonic crust (Case I)
  - Mainly nucleonic matter with small quark core (Case II)



- Effect of CSC very small.
- Hyperonic/Kaonic EoS ruled out.
- For neutron and hybrid stars, $M \lesssim 2M_{\text{solar}}$.
- Cannot exclude very large stars in case of stable strange quark matter
  - Dense quark stars ruled out!





- Maximal mass independent of the maching point
- CSC increases pressure ⟶ smaller stars

Conclusions

- The grand potential of QCD at finite density with finite m_s computed to α_s^2.
  - To tackle technical challenges needed to create new perturbation theory machinery.
- Modeled the EoS in the full range of μ (three logical possibilities):
  1. Hardon / quark matter transition
     - A realistic description of thermodynamics on all values of μ
  2. Absolutely stable strange quark matter
     - is disfavored, but not ruled out
     - ...but the observation of stars $M > 2M_{\mathrm{solar}}$ might be a strong evidence in the opposite direction.
  3. Exotic (non-CSC) phases between hadrons and quark matter
     - Well, at least me improved the perturbative side. . .

Outlook

- Improve the modelling of CSC:
  - Computing the mismatch of the fermi spheres
  - Assessing the different possibilities for CSC: CFL, 2CS...
- Improve the perturbative calculation:
  - $\alpha_s^3 \log(\alpha_s)$: only ring diagrams involved.
  - α_s^3: Major undertaking.
- Improve the compact star calculations:
  - Two-component mixtures of hadronic and quark matter
  - Moment of inertia, glichtes
  - Neutron star oscillations
  - Rotating stars, r-modes
  - Cooling rates and transport effects
  - ...

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    - …
- …and of course:
  the observations are advancing very fast, new data to come!!

Hyvää Joulua!