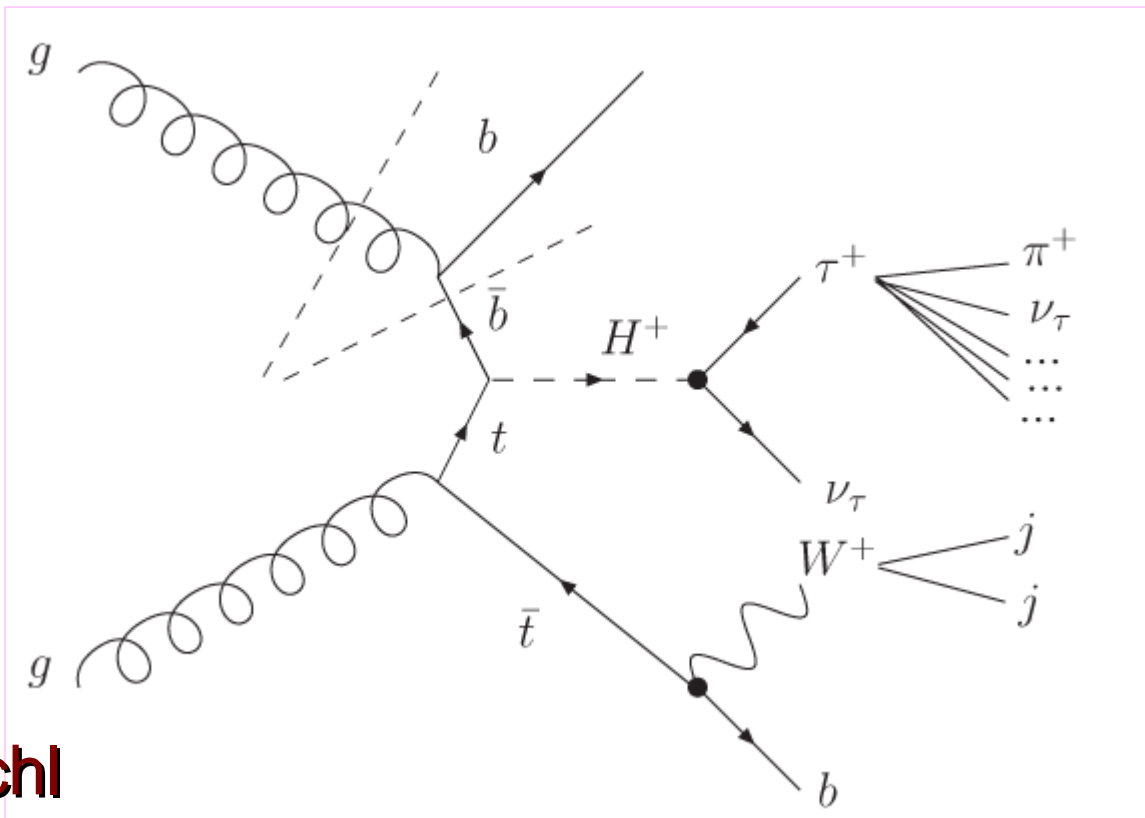




# Charged Higgs Boson Searches at ATLAS



**Martin Flechl**  
 (Institutionen för Fysik och Astronomi, Uppsala)





# Outline

1. Charged Higgs Boson: 1-page-Introduction
2. Heavy  $H^+ \rightarrow \tau\nu$  Searches (2007):  
ATLAS-PHYS-PUB-2007-006  
(Bjarte Mohn/MF/Johan Alwall)
3. Current ATLAS  $H^+$  Efforts (CSC)  
Six channels, currently in review  
-- Preliminary --



# The Charged Higgs Boson

- **Standard Model (SM) Higgs Boson:**

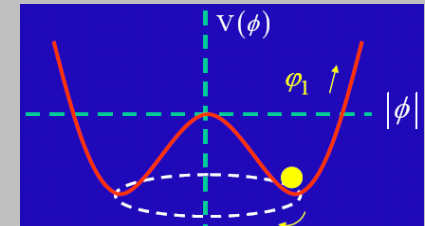
- Explicit **mass terms** in the SM Lagrangian **break gauge invariance**

- possible **solution**: Complex scalar doublet and a potential with a non-zero vacuum expectation value: => **Higgs Mechanism:**

- effective mass terms for  $W^\pm$ ,  $Z$ ;

- fermions obtain mass by coupling to the Higgs field;

- one physical particle: The **SM Higgs Boson**

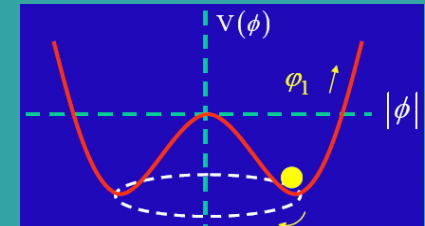




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- **Supersymmetry (SUSY):**

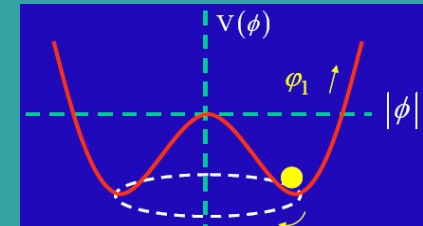
- Hierarchy problem: SM Higgs sector is **unstable: radiative corrections** to the Higgs boson mass
- **Supersymmetry** stabilizes divergent loop contributions to the Higgs mass by introducing **new fermions and bosons** (superpartners to SM-particles)
- additional benefits: coupling constant unification, DM Candidate, SUGRA, Superstrings...



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- **Minimum SUSY Higgs Sector: Two Higgs Doublet Model (THDM):**

- In SUSY: **Two Higgs doublets necessary** to give mass to all fermions without breaking gauge invariance: => **five physical Higgs bosons:**
  - $h^0$  (SM-like light, neutral),  $A^0$  (neutral, CP-odd),  $H^0$  (neutral, CP-even)
  - $H^\pm$ : charged, mass  $\approx 10^2$ - $10^3$  GeV => SUSY accessible below  $M_{\text{SUSY}}$ !
- at tree level: described by two parameters, e.g.  $m_{H^\pm}$  and  $\tan \beta$  (ratio of Higgs doublet VEVs)



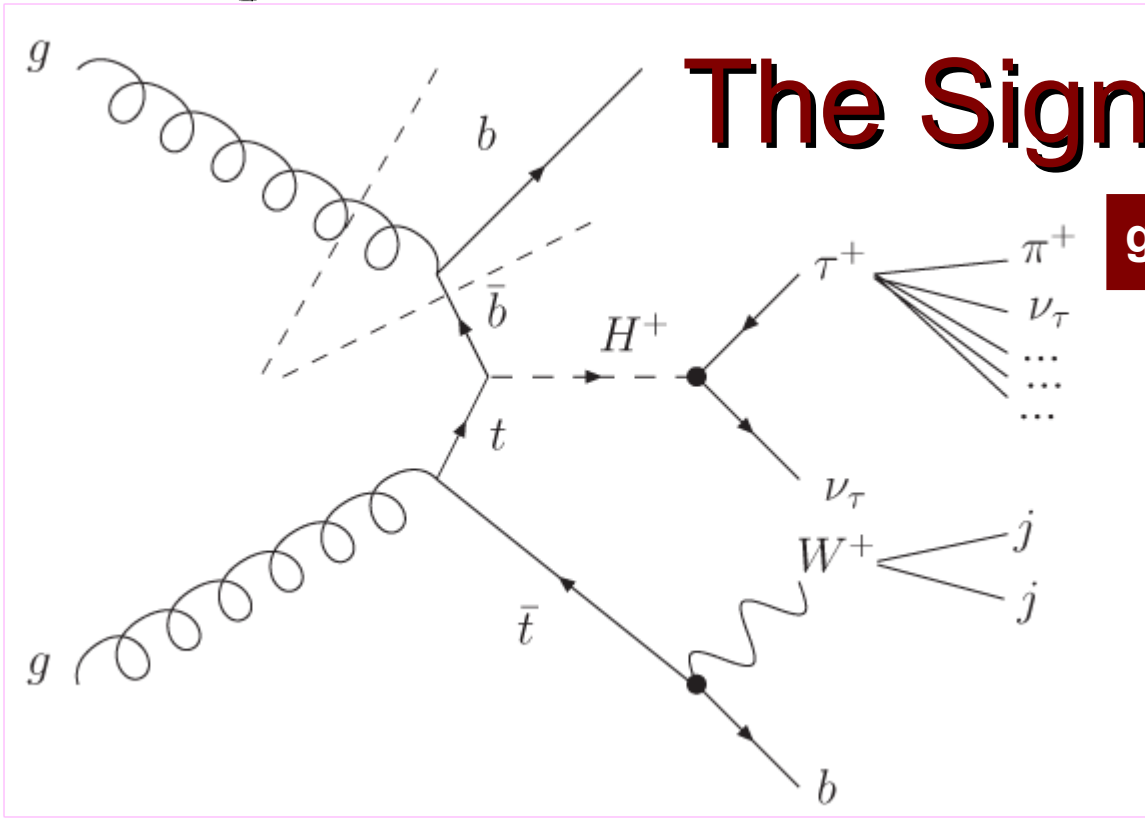
# 1. heavy $H^+ \rightarrow \tau \nu$ (2007)

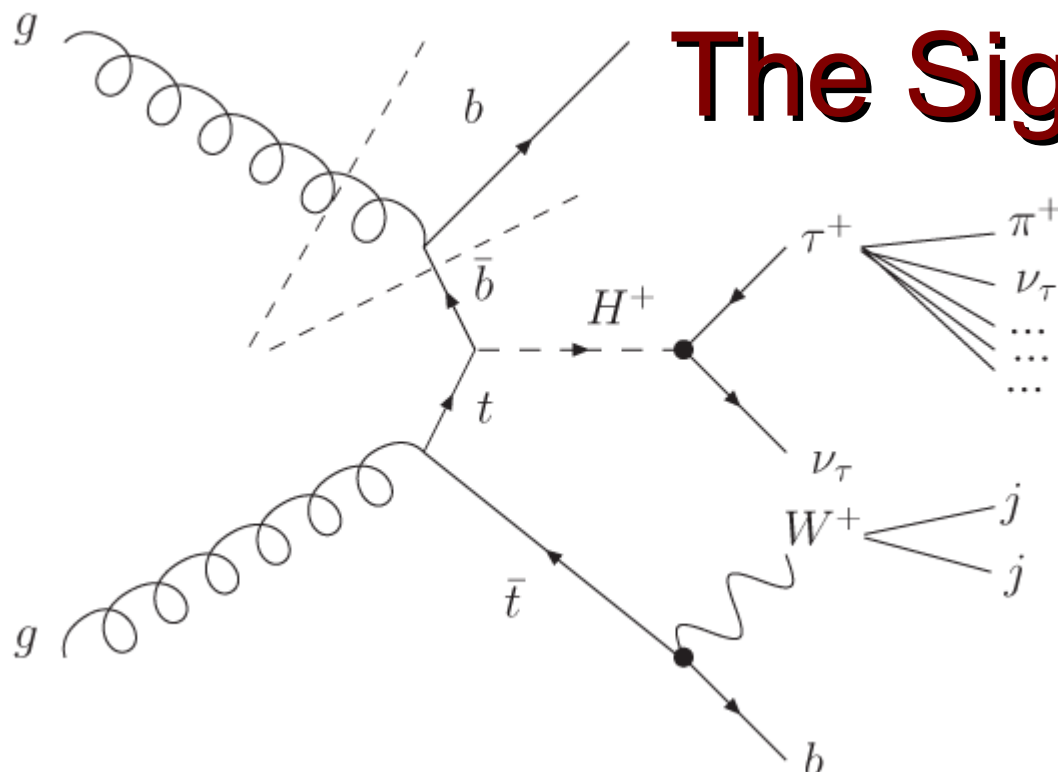
Mohn/Flechl/Alwall, ATLAS-PHYS-PUB-2007-006



# The Signal

$gb/gg \rightarrow H^+ t [b] \rightarrow (jjb) (\tau \nu_\tau) [b]$





# The Signal

$gb/gg \rightarrow H^+ t [b] \rightarrow (jjb) (\tau \nu_\tau) [b]$

**Production:**

Sample	MC Events
$H^+$ 165 GeV	$10^4$
$H^+$ 175 GeV	$10^4$
$H^+$ 200 GeV	$10^4$
$H^+$ 300 GeV	$10^4$
$H^+$ 400 GeV	$10^4$
$H^+$ 500 GeV	$10^4$
$H^+$ 600 GeV	$10^4$

- **Signature:** hadronically decaying  $\tau$ ; missing energy  $\nu_\tau$ ; top:  $t \rightarrow bjj$
- **Event Generation:** Matched production  $gg \rightarrow tbH^+ / gb \rightarrow tH^+$  (hep-ph/0409094)  
Event Generators: Matchig  $\rightarrow$  Pythia  $\rightarrow$  Tauola/Photos
- **Detector simulation:** ATLAS **Full Simulation** (Athena 9.04, Reconstruction: 11.0.4)
- **Number of events:** **10 000 per mass** (165, 175, 200, 300, 400, 500, 600 GeV)



# The Backgrounds

•Detector simulation: ATLAS **Fast Simulation** (Atlfast)

**W + jets**  
(AlpGen)

**Single Top (Wtb)**  
(TopRex)

## QCD dijet

- signature: no hard  $\tau$ , no large  $p_T^{\text{miss}}$
- LHC: high cross section- $O(10^{12} \text{ fb}^{-1})$

## $t\bar{t}$ : Main Background

- $t\bar{t} \rightarrow (\tau\nu b)(j\bar{j}b)$  resembles signal signature, but all decay modes were simulated



# The Backgrounds

- **Event Generation:** ... + Pythia
- **Detector simulation:** ATLAS **Fast Simulation** (Atlfast)

Sample (ckin12)	MC Events
QCD 20-50	$4 \cdot 10^7$
QCD 50-100	$4 \cdot 10^7$
QCD 100-200	$4 \cdot 10^7$
QCD 200-300	$4 \cdot 10^7$
QCD 300-400	$9 \cdot 10^7$
QCD 400-500	$3 \cdot 10^7$
QCD 500-600	$4 \cdot 10^7$
QCD 600-1000	$4 \cdot 10^7$
QCD 1000-10000	$4 \cdot 10^7$

## QCD dijet

- signature: no hard  $\tau$ , no large  $p_T^{\text{miss}}$
- LHC: high cross section- $O(10^{12} \text{ fb}^{-1})$
- a jet-weighting technique was used to increase effective sample size (see ATL-COM-PHYS-2006-066)

Sample	MC Events
W+1jet	$1.2 \cdot 10^8$
W+2jets	$1.4 \cdot 10^7$
W+3jets	$2.9 \cdot 10^6$
W+4jets	$5.0 \cdot 10^5$
W+5jets	$1.3 \cdot 10^5$

**W + jets**  
(Alpgen)

Wtb (tau)	$1.1 \cdot 10^6$
-----------	------------------

**Single Top (Wtb)**  
(TopRex)

ttbar	$1.0 \cdot 10^8$
-------	------------------

## ttbar: Main Background

- Large sample: corresponds to  $\approx 200 \text{ fb}^{-1}$ .
- $tt \rightarrow (\tau\nu b)(j\bar{b})$  resembles signal signature, but all decay modes were simulated



# Analysis: The Selection Cuts

→ **Three sets** of selection cut values:

Optimum cut values: Correlated to  $m_{H^\pm}$ .

Different values are used for

I )  $(165 < m_{H^\pm} < 250)$  GeV

II )  $(250 < m_{H^\pm} < 450)$  GeV

III)  $(450 < m_{H^\pm} < 600)$  GeV



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→ applied **selection cuts (Three new ones)**:

## 1. Jet Identification, Lepton Veto

$1\tau$ ;  $\geq 3$  jets, one and only one b-tagged; **Lepton Veto** (7 GeV)

$p_{T\tau} > 40$  GeV,  $E_{T\text{miss}} > 40$  GeV.

## 2. W and Top Reconstruction

a) W, b) top reconstructed, within 25 GeV mass-windows

## 3. Tau-Cuts

$p_T^\tau < [65, 80, 100]$  GeV,  $\eta^\tau < [0.9, 1.0, 1.2]$ ,  **$p_T$ -Ratio**  $< [6.0, 5.5, 5.0]$

## 4. $p_{T\text{miss}}$

$p_T^{\text{miss}} < [120, 135, 165]$

## 5. DeltaPhi - $\angle(\tau, p_T^{\text{miss}})$

$\Delta\phi > [1.1, 1.2, 1.3]$



# Results (1)

MSSM mh-max;  
no trigger, no  
systematic errors

$m_{H^+} = 165 \text{ GeV}$

$m_{H^+} = 300 \text{ GeV}$

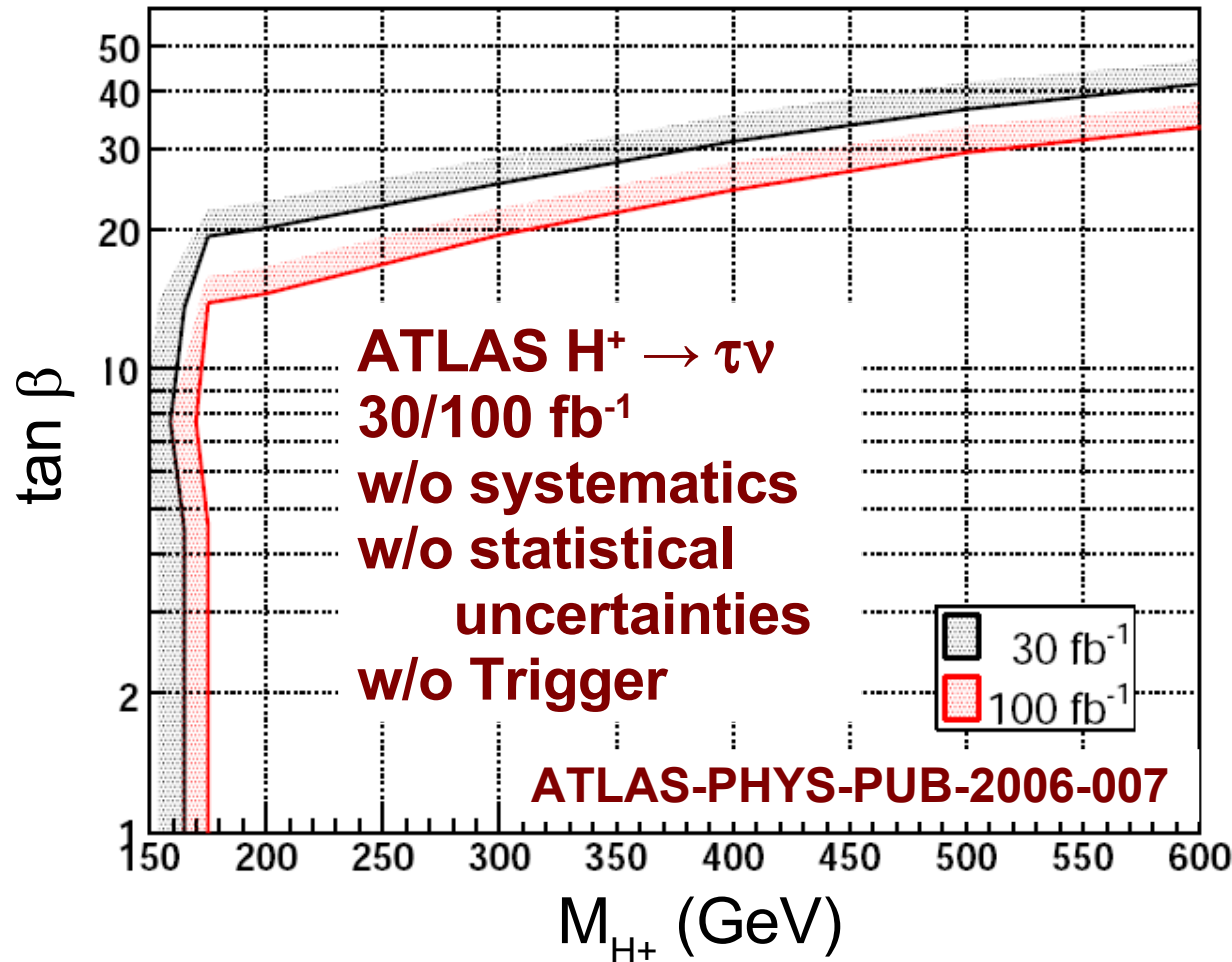
Sample	events/30fb <sup>-1</sup> , tan β=35		S/√B
	signal	ttbar	
All events	5.6 · 10 <sup>4</sup>	1.8 · 10 <sup>7</sup>	0.01
Precuts	2500	24000	15
Tau-Cuts	900	1300	22
Pt miss	500	700	19
<b>DeltaPhi</b>	<b>239</b>	<b>39</b>	<b>35.9</b>

Sample	events/30fb <sup>-1</sup> , tan β=35		S/√B
	signal	ttbar	
All events	2.6 · 10 <sup>3</sup>	1.8 · 10 <sup>7</sup>	0.0005
Precuts	200	24000	1.1
Tau-Cuts	70	750	2.4
Pt miss	30	150	2.5
<b>DeltaPhi</b>	<b>26</b>	<b>2</b>	<b>18.1</b>

- **Medium / High mass range**: depend on hard  $p_T^\tau$  and  $p_T^{\text{miss}}$  cuts (small  $\sigma$ ), almost all background can be suppressed
- **Low mass range**: Discovery only possible for large signal cross section (as in the MSSM) due to irreducible background
- **Efficiency**:  $\approx 4$ -6 orders of magnitude higher for signal than for ttbar background



# Results (2)



- region  $m_{H^+} < 160$  GeV covered
  - Discovery reach up to high  $H^+$  masses for high  $\tan\beta$
  - an important leap in “realism” - Fullsim signal and all relevant backgrounds considered
- BUT:**
- very optimistic wrt to real data:
    - no systematics treatment
    - no trigger simulation
    - no pile-up
    - Fastsim background
    - Gaussian approximation



## 2. Ongoing ATLAS $H^+$ activities ( $H^+$ CSC Note - in review)



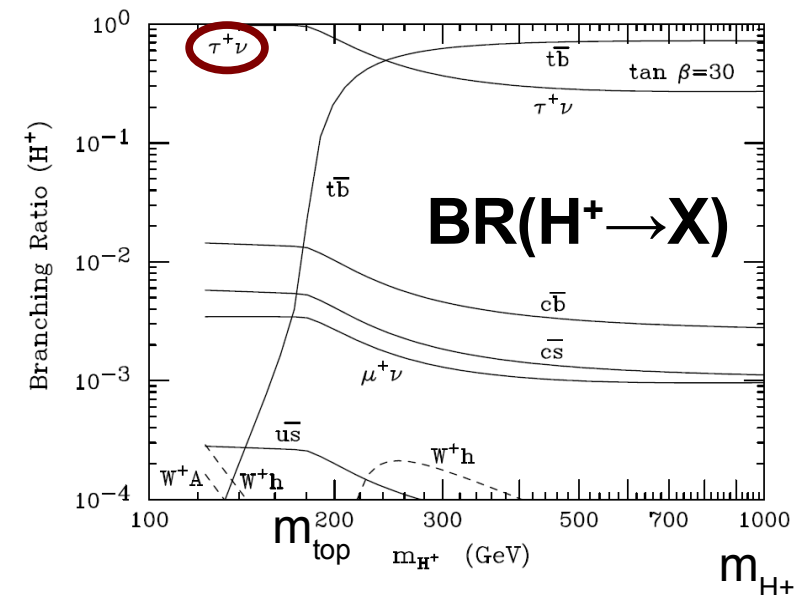
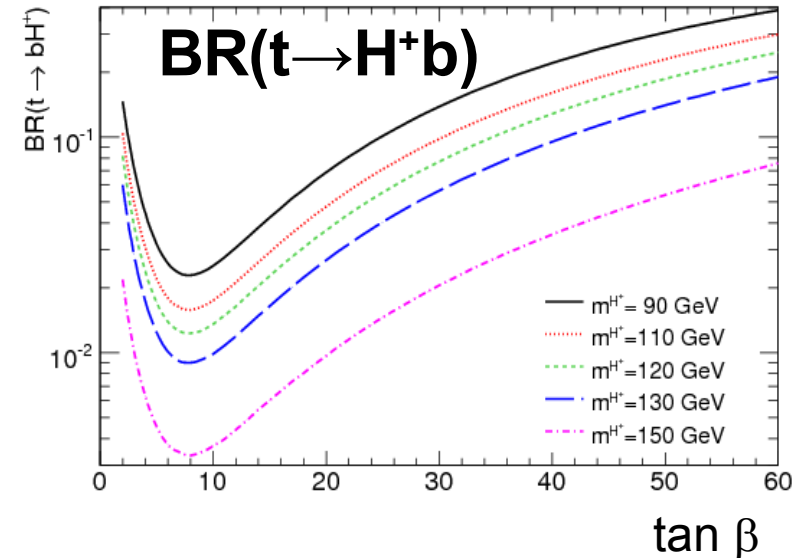


$$m_{H^+} < m_{\text{top}}$$

- Production: top quark decays, via  $t\bar{t} \rightarrow bW^- bH^+$
- $\text{BR}(t \rightarrow bH^+)$  depends on  $m_{H^+}$  and  $\tan \beta$

- $\text{BR}(H^+ \rightarrow \tau\nu) \approx 1$   
→ Main channels of interest:

$H^+$ Decay	W Decay
$\tau\nu, \tau \rightarrow \text{had}$	qq
$\tau\nu, \tau \rightarrow \text{had}$	lv
$\tau\nu, \tau \rightarrow \text{lep}$	qq

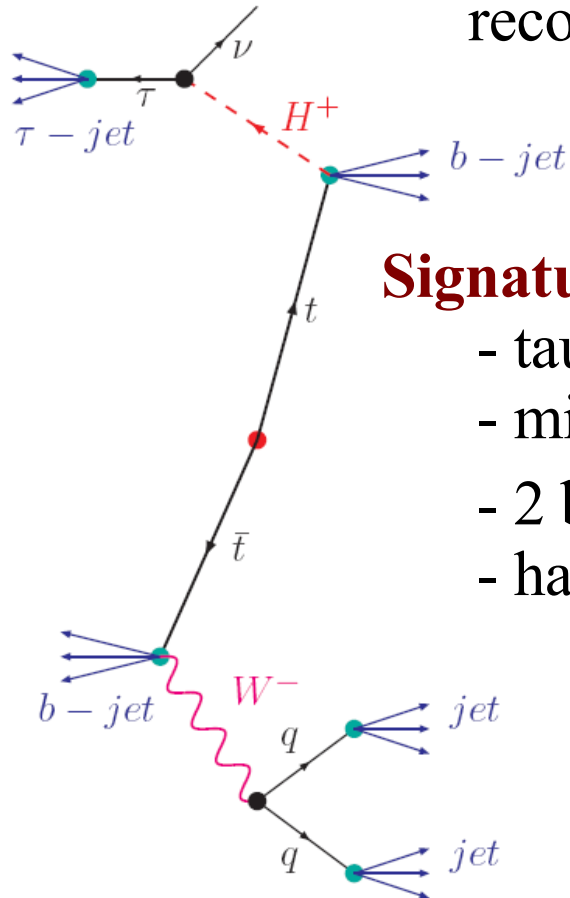




$$H^+ \rightarrow \tau_{\text{had}} \nu, W \rightarrow jj$$

Elias Coniavitis (Uppsala)

- in the last years main ATLAS focus for  $H^+$  with early data
- $H^+$  Transverse Mass can be reconstructed



### Signature:

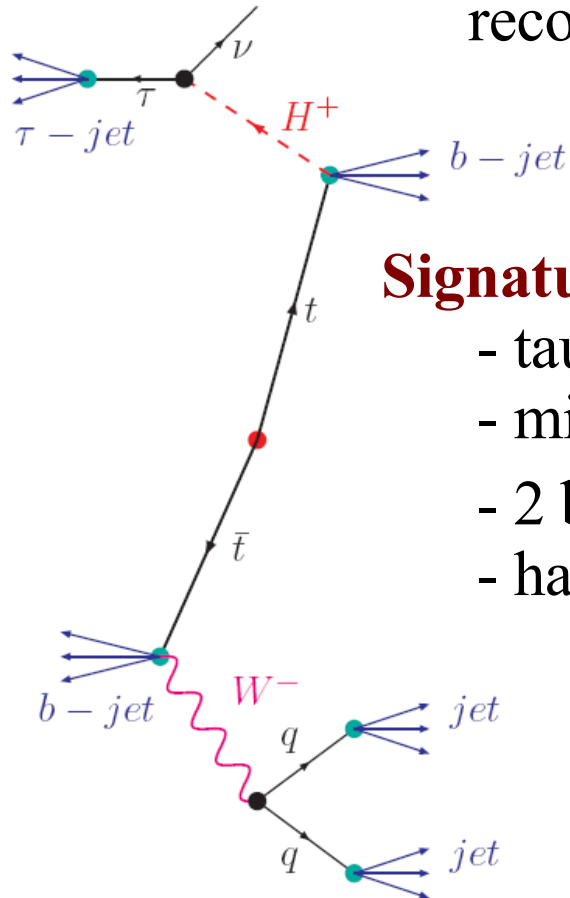
- tau-jet
- missing  $E_T$
- 2 b-jets
- hadronic W/top



$$H^+ \rightarrow \tau_{\text{had}} \nu, W \rightarrow jj$$

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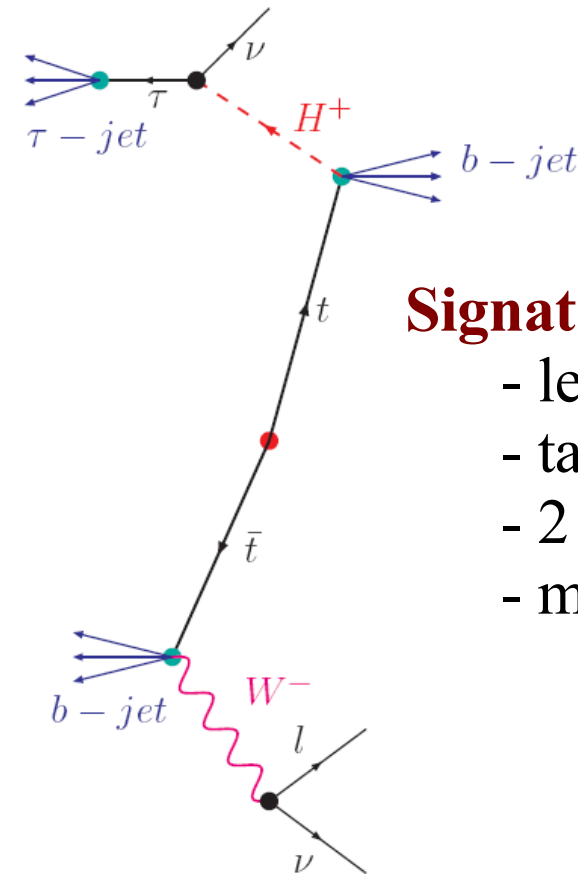
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$$H^+ \rightarrow \tau_{\text{had}} \nu, W \rightarrow l \nu$$

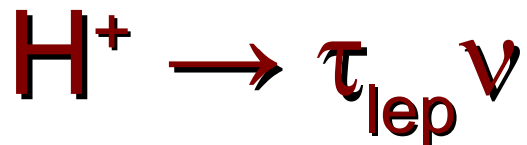
(A. Ahmad, T. Ehrich, S. Mohrdieck-Moeck)

- CMS: “Prime  $H^+$  Discovery Channel”
- revisited for ATLAS after 15 years
- counting experiment



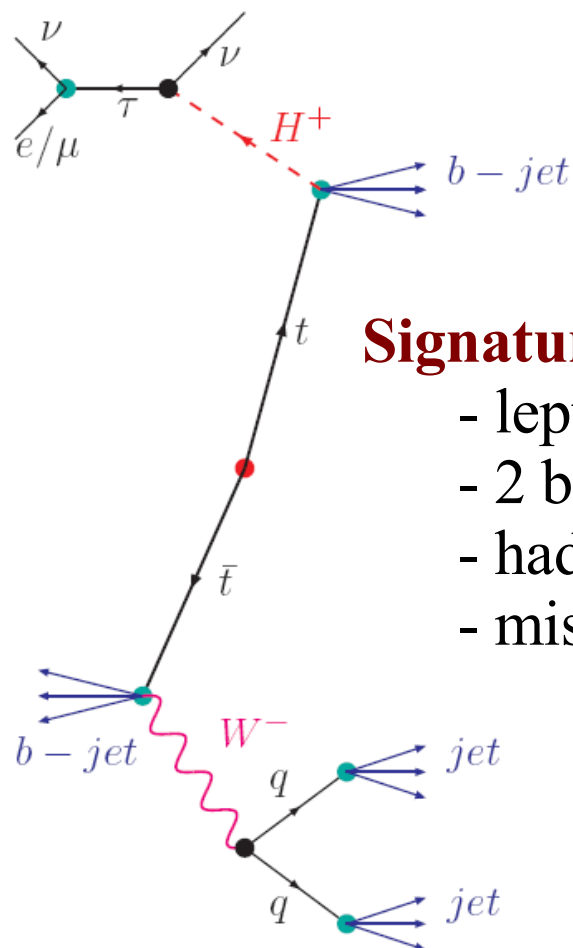
### Signature:

- lepton
- tau-jet
- 2 b-jets
- missing  $E_T$



Ofer Vitells, Eilam Gross (Weizmann)

- First ATLAS investigation
- Novel  $m_T$ -technique



### Signature:

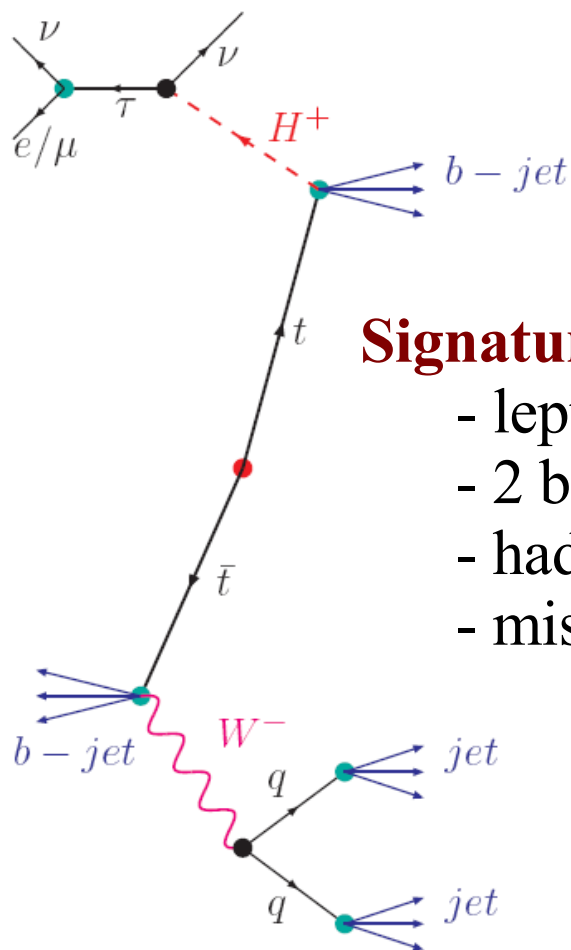
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- missing  $E_T$



$$H^+ \rightarrow \tau_{\text{lep}} \nu$$

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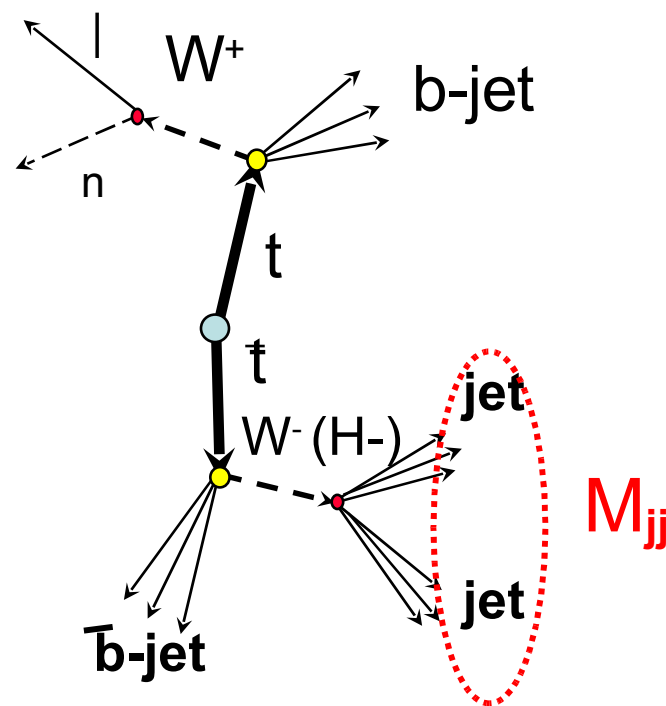
### Signature:

- lepton
- 2  $b$ -jets
- hadronic  $W$ /top
- missing  $E_T$

$$H^+ \rightarrow CS$$

(J. Lane, F. Loebinger, P.S. Miyagawa, J. Pater, U.K. Yang)

- Uses CDF Likelihood fitter on the dijet-mass distribution
- look for second “bump” in the mass distribution (first one due to  $W$ )



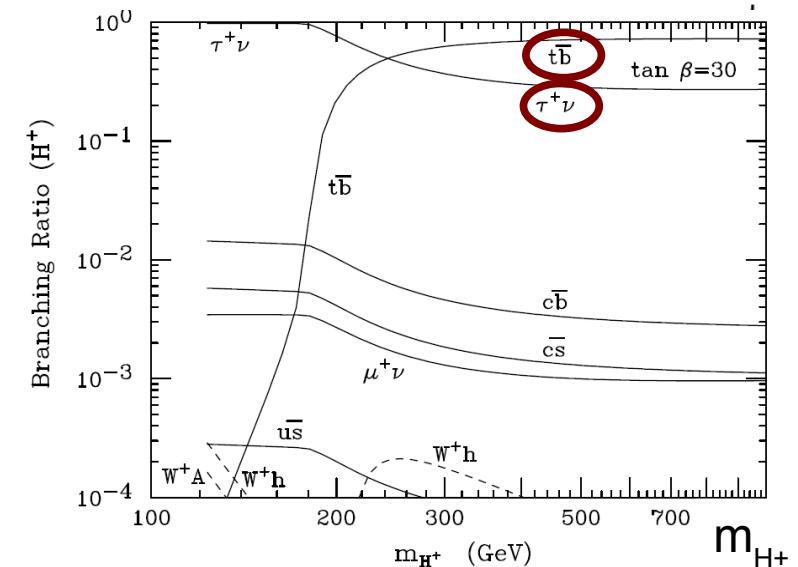
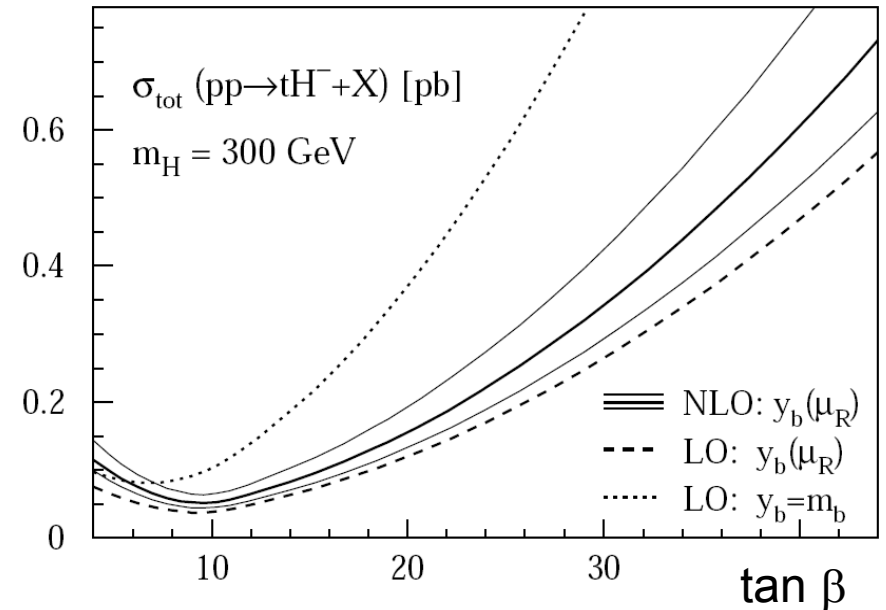


$$m_{H^+} > m_{\text{top}}$$

- Production: **gg/gb-fusion**  
 $gg \rightarrow tbH^+$ ,  $gb \rightarrow tH^+$   
 (→double-counting term, resolved by MC Generator “Matchig”)  
 - cross section much smaller than in  $t\bar{t}$  decays

- **$BR(H^+ \rightarrow t\bar{b}) \approx 0.8-1$**   
 → Main channels of interest:

$H^+$ Decay	W Decay
$\tau\nu, \tau \rightarrow \text{had}$	$qq$
$t\bar{b}$	$lv \ \& \ qq \ (2 \ Ws)$



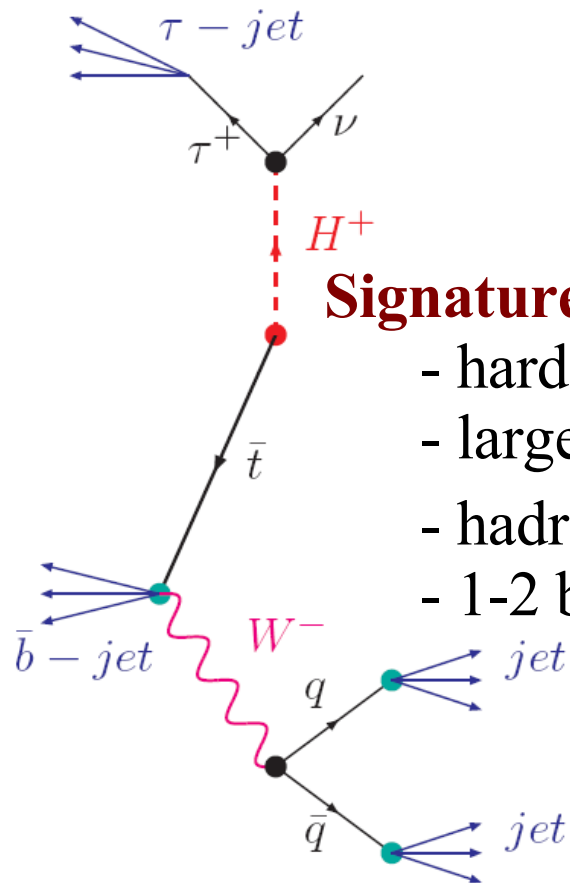




# heavy $H^+ \rightarrow \tau_{\text{had}} \nu$

Bjarte Mohn (Bergen), MF

- Clean: High missing  $E_T$  allows reduction of QCD background
- Small  $\text{BR}(H^+ \rightarrow tb) \approx 1$



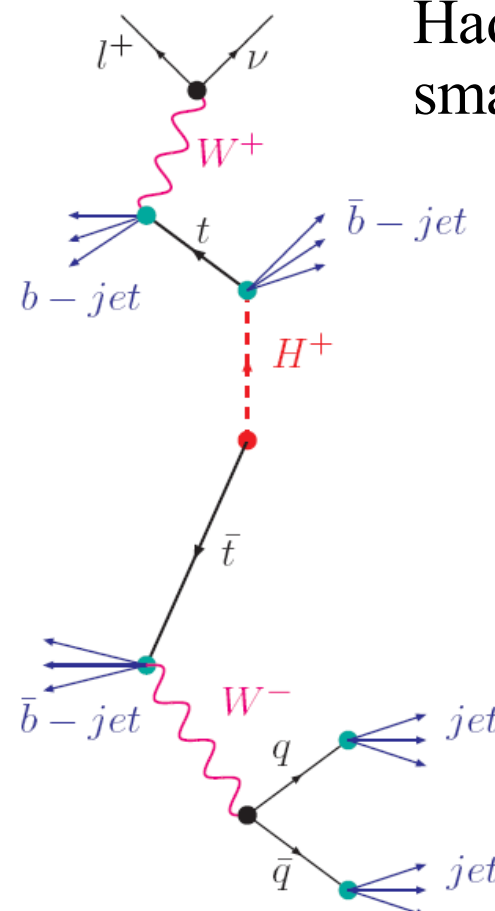
## Signature:

- hard tau-jet
- large missing  $E_T$
- hadronic W, top
- 1-2 b-jets

# heavy $H^+ \rightarrow tb$

Remy Zaidan, Mossadek Talby (Marseille)

- Heavy:  $m_{H^+} > m_{\text{top}}$
- Big  $\text{BR}(H^+ \rightarrow tb) \approx 1$
- Dirty: Hadronic decays at a Hadron Collider, small missing  $E_T$



## Signature:

- 3-4 b-jets
- lepton
- 2 W, 2 top



# Results



# (Instead of) Results

- *Sorry, no results/discovery contours today* (review process ongoing)
- **First  $H^+$  Fullsim results for ATLAS** (together with 2007 study), some channels investigated for the first time at all
- Apart from results, **an important experience in many respects**:
  - social exercise: first coordinated collaboration, homogeneous framework/techniques/visualisation/...
  - uncovered several problems, such as
    - importance of large  $t\bar{t}$  samples (statistical error)
    - trigger menus develop – threaten low  $H^+$  channels
  - first time rigorous treatment of
    - systematic & statistical uncertainties
    - trigger, pileup
    - background estimation from data



# Summary & Outlook

- The **ATLAS discovery potential for  $H^\pm$  decaying to  $\tau \nu_\tau$**  has been shown as studied in ATLAS-PHYS-PUB-2007-006
- An update is immanent and will be approved within the following months:
  - “**ATLAS CSC Charged Higgs Note**”, employing a more rigorous treatment, investigating several ATLAS  $H^\pm$  channels with Fullsim for the first time
- An interesting time is waiting for us
  - $H^\pm$  detection can give handle on SUSY even for high  $M_{\text{SUSY}}$
  - for special cases (low  $m_{H^\pm}$ , high  $\tan \beta$ ) an early discovery is possible (or exclusion surpassing Tevatron)



# Backup Slides

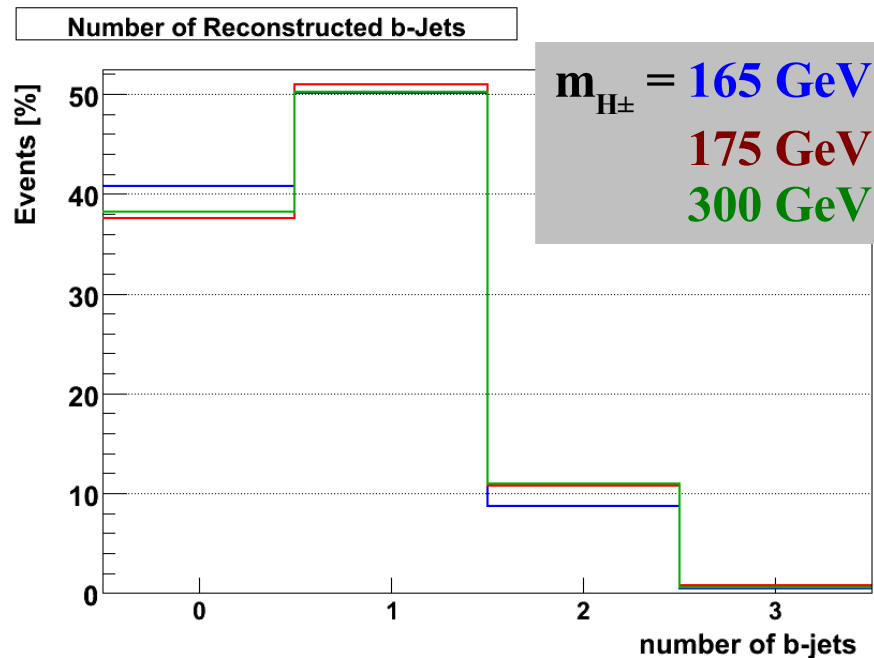
- ATLAS-PHYS-PUB-2007-006:
  - Selection Cuts: Plots, Efficiencies, ...
  - Parameter space scans, b- and tau-tagging strategies
- CSC Note:
  - Transverse Mass-technique



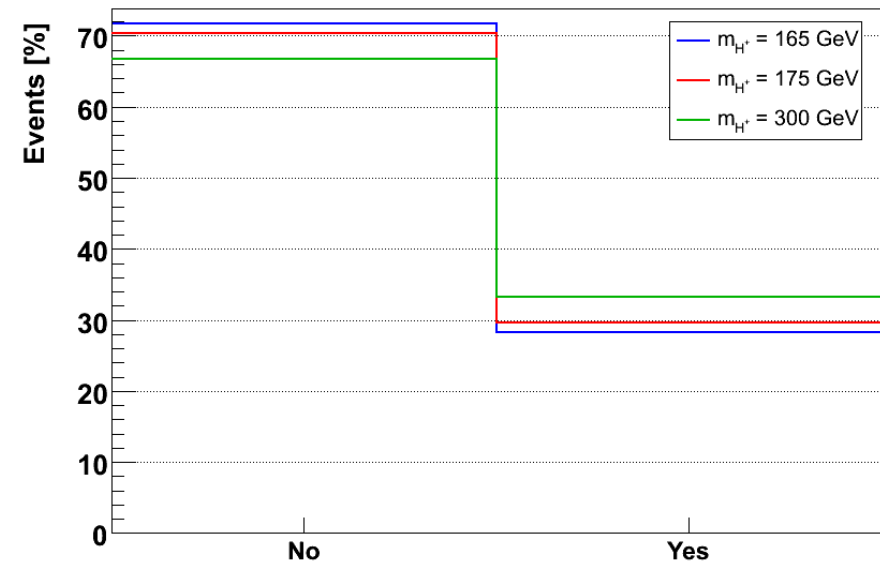
# Cuts: a) Jets+LV (1)

**Cut: exactly one b-jet reconstructed**

$gg \rightarrow tbH^\pm$  : 2 b-jets  
 $gb \rightarrow tH^\pm$  : 1 b-jet (visible)



**Tau Lepton Identification Efficiency**



**Cut: exactly one  $\tau$ -jet reconstructed**

$\tau$ -tagging efficiency: 45%

30-35%  $\tau$ -identification efficiency



# Cuts: a) Jets+LV (2)

ttbar events surviving all standard cuts (30fb<sup>-1</sup>):

W <sub>1</sub>	W <sub>2</sub>	Standard Cuts	Lepton Veto
lepton	lepton	0,9	0,4
lepton	tau	48,2	17,7
lepton	jets	27,6	9,2
jets	tau	24,0	19,5
jets	jets	0,7	0,7
tau	tau	42,8	35,4
<i>all</i>		144,2	83

Annotations:

- Red arrows point to the top three rows (lepton channels).
- Blue arrow points to the row (jets, tau) labeled "similar to signal".
- Green arrow points to the row (tau, tau) labeled "largest contribution".

**Two surprising results** for  $tt \rightarrow (W_1 b)(W_2 b)$ : - >50% of whole background: lepton channels  
- dominant: decay with two tau-jets

**Cut: Veto on a Lepton  
with  $p_T > 7$  GeV**

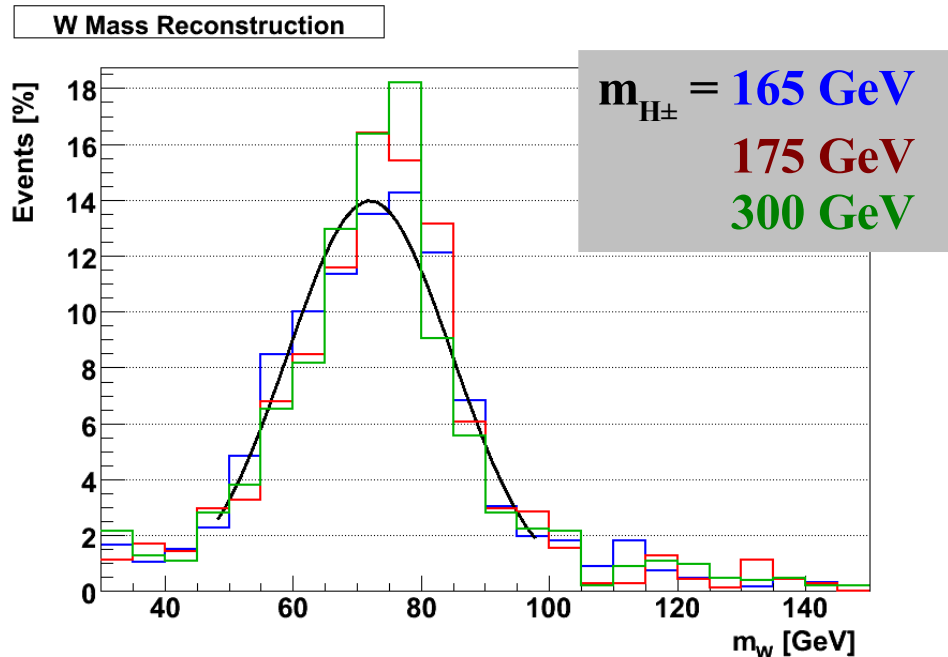
**Other Cuts:  $\geq 3$  parton jets  
 $p_T^\tau > 40, p_T^{\text{miss}} > 40$**

- 2/3 of ttbar events with **leptonic decays** suppressed
- additionally, 20% of ttbar events with **decays to  $\tau$**
- **signal** and other backgrounds (QCD, Wtb, W+jets) **almost unaffected**



# Cuts: b) W- & Top-Reconstruction

Cut: W candidate with  $|m_{jj}-73| < 25$  (GeV)

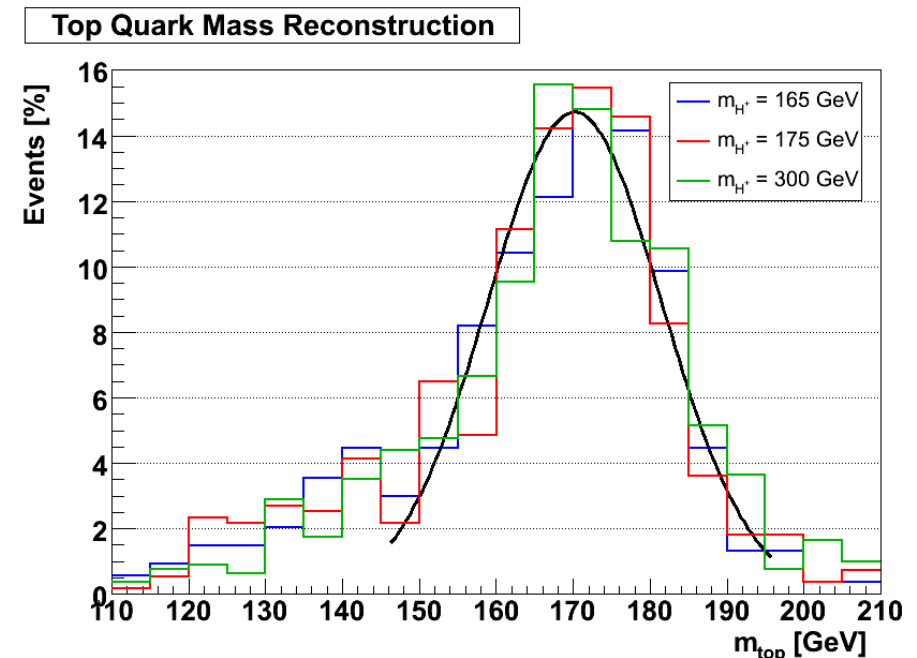


- reserve one b-jet
- find **jet-pairs** which satisfy  $|m_{jj}-73| < 25$

known jet-Miscalibration  
in the ATLAS software  
release used

- all W candidates which passed are kept
- jet-four momenta rescaled to  $m_W=80.14$
- **b-jets added**, tested for  $|m_{bjj}-171| < 25$

Cut: top candidate with  $|m_{bjj}-171| < 25$  (GeV)





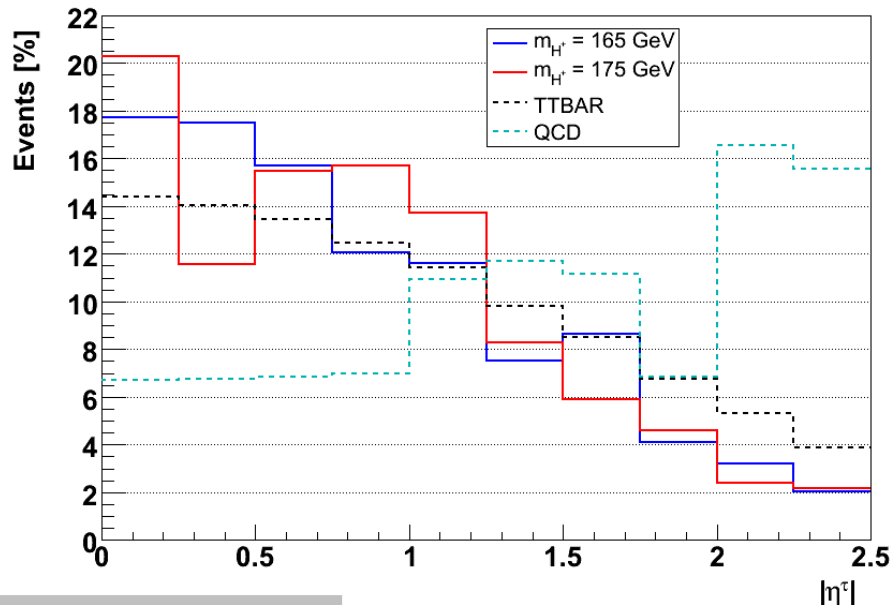
# Cuts: c) Tau Lepton (1)

Transverse Momentum → P<sub>T</sub>-Ratio (to hardest non-top-jet)  
Transverse Momentum

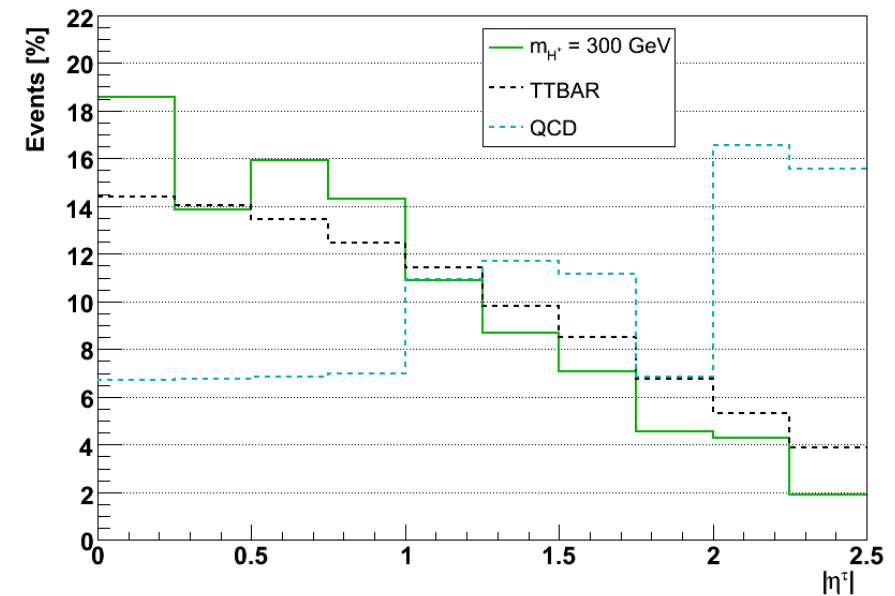
Aim: Replace p<sub>T</sub>-cut  
by a set of cuts  
performing equally  
well for all m<sub>H±</sub>

**Cut: Tau Pseudorapidity  $\eta^\tau > [1.1; 1.2; 1.3]$**

Tau Lepton Pseudorapidity (Low Mass Region)



Tau Lepton Pseudorapidity (Medium Mass Region)



$m_{H^\pm} = 165 \text{ GeV}$   
 $175 \text{ GeV}$   
 $300 \text{ GeV}$

- **fake  $\tau$ -jets**: high mistagging-rate for large  $\eta$  (QCD)
- **real  $\tau$ -jets**: more central for signal than for ttbar

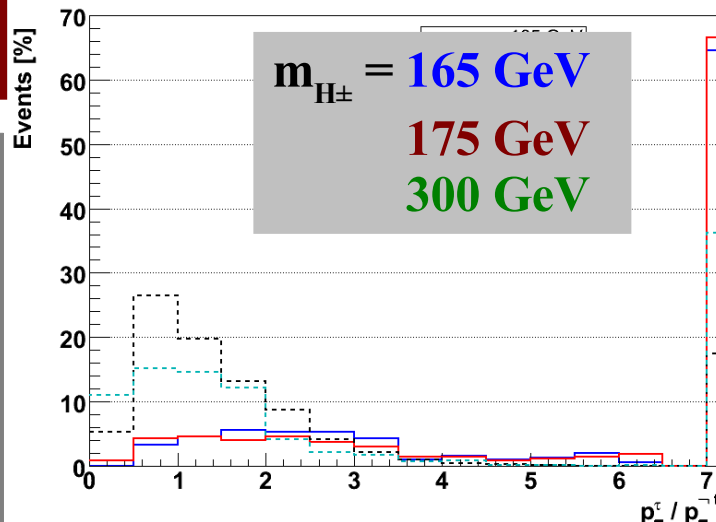


# Cuts: c) Tau Lepton (2)

**Cut:**  
 $P_T\text{-Ratio} > [3.2; 5.0; 5.0]$

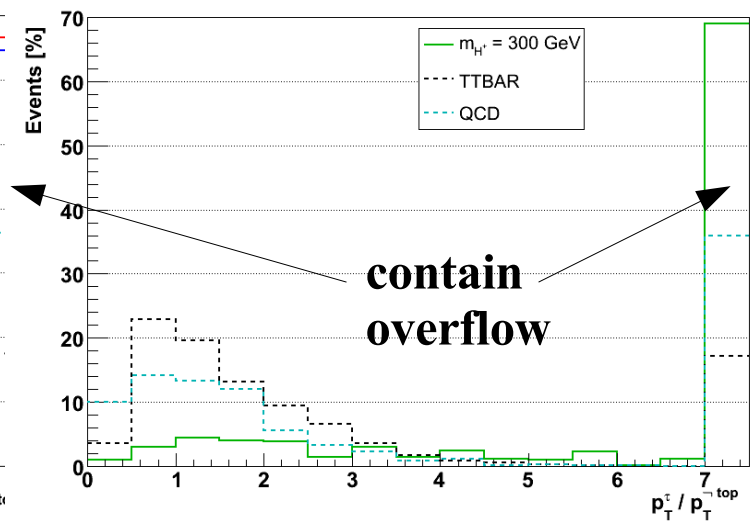
- $P_T$ :  $\tau$  to hardest parton jet not used for top-Reco
- **low**  $m_{H^\pm}$ : no hard additional jet
- **high**  $m_{H^\pm}$ :  $\tau$  very hard
- **ttbar**: additional jets

Ratio of Transverse Momenta (Low Mass Region)



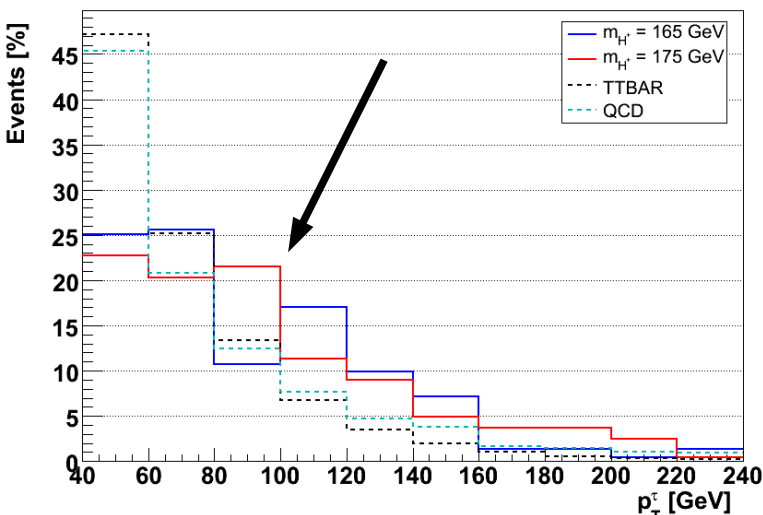
efficiency: signal  $\sim 80\%$ ; background  $\sim 30\text{-}40\%$

Ratio of Transverse Momenta (Medium Mass Region)

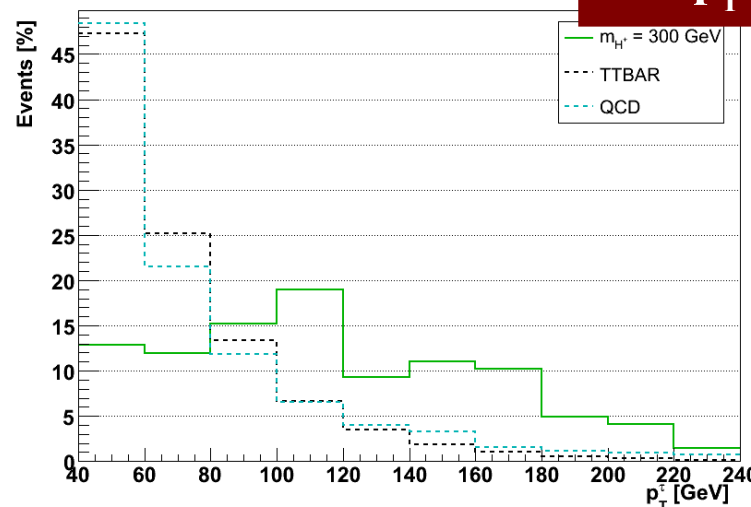


**Cut: Tau Transverse Momentum**  
 $p_T^\tau > [65; 80; 100] \text{ GeV}$

Tau Lepton Transverse Momentum (Low Mass Region)



Tau Lepton Transverse Momentum (Medium Mass Region)

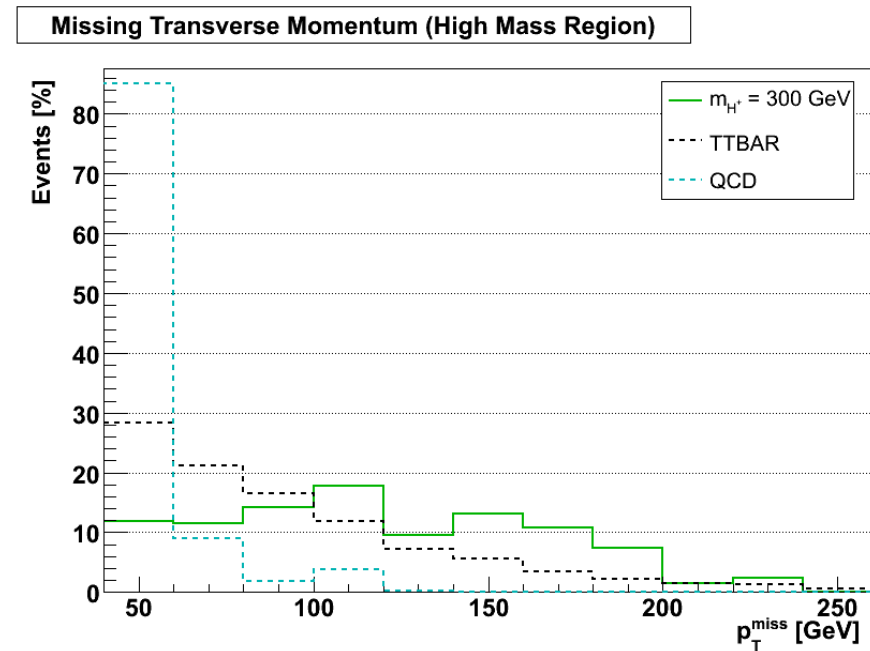
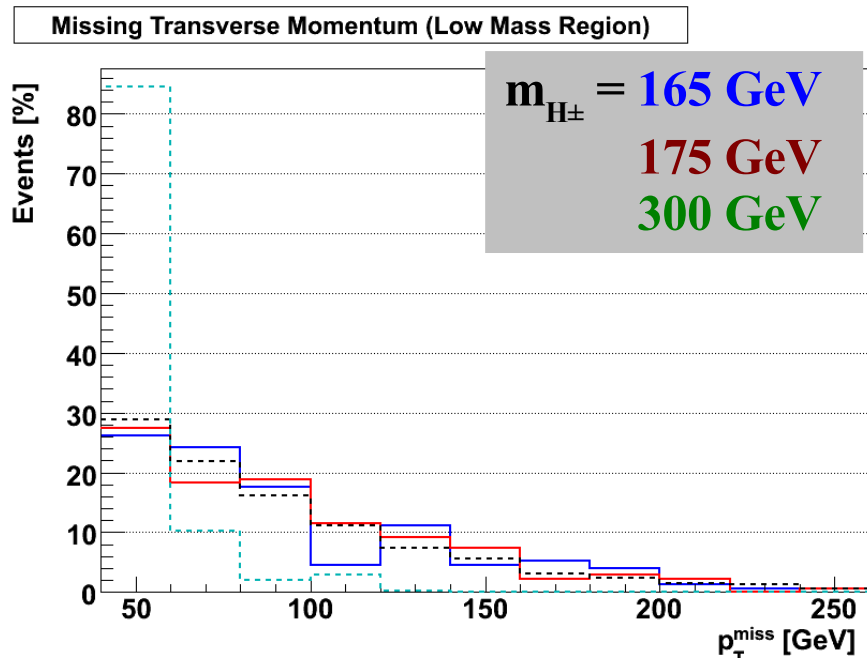


- $P_T$ -Ratio cut allows softer  $p_T^\tau$ -cuts
- **New set of  $\tau$ -cuts (for  $165 < m_{H^\pm} < 250$ ):**  
**4x more signal,**  
**4x less Background**



# Cuts: d) Missing $p_T$

Cut: Missing Transverse Momentum  $p_T^{\text{miss}} > [75; 130; 145]$  GeV



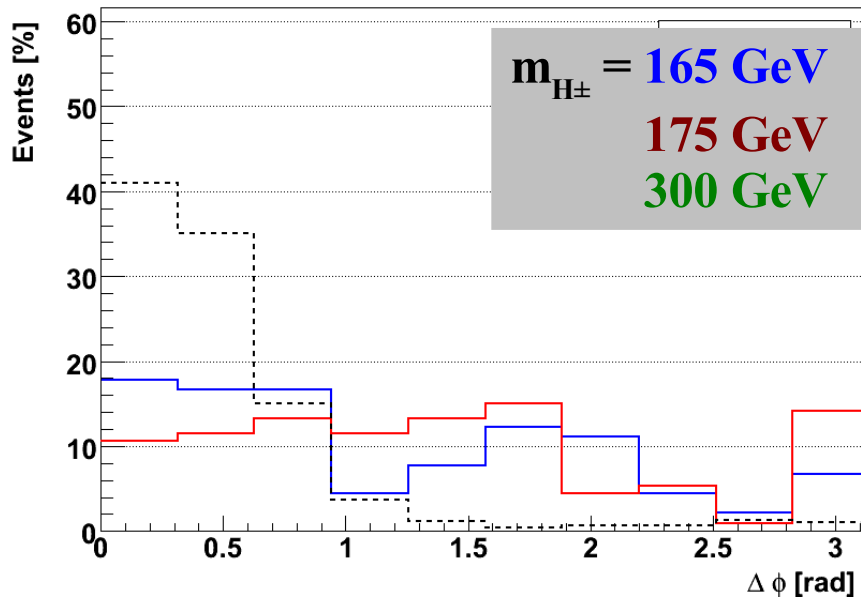
- **QCD**: high rejection (no real missing  $p_T$ )
- **ttbar**: **efficiency** between **15%** (high mass) and **50%** (low mass)
- **signal**: efficiency about **60%**  
 → optimum cut value highly correlated to next cut:  $\Delta\phi$



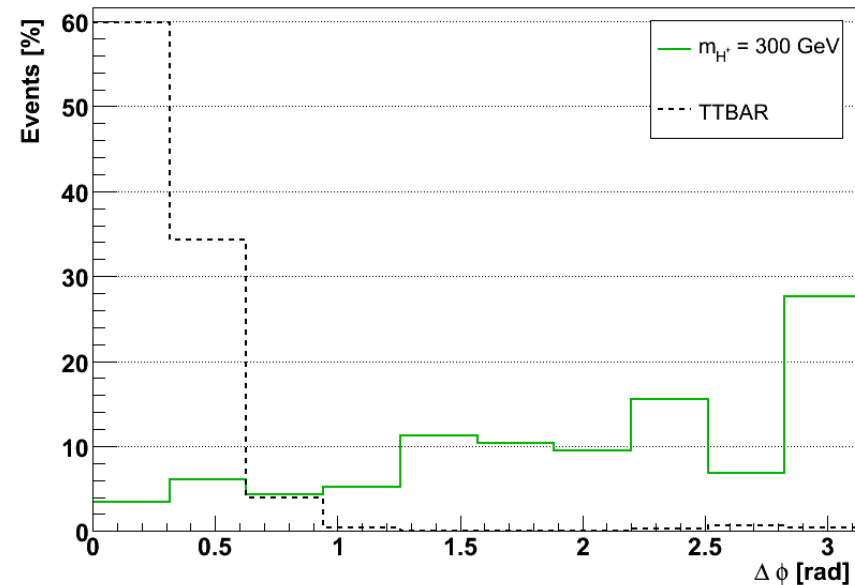
# Cuts: e) Azimuthal Opening Angle

Cut: Azimuthal Opening Angle  $\Delta\phi > [1.2; 1.2; 0.9]$

Azimuthal Opening Angle (Low Mass Region)



Azimuthal Opening Angle (High Mass Region)



## Motivation:

- **Backgrounds:** Only  $\tau$  and  $\nu$  from a  $W$  (80 GeV) with a **large  $p_T$ -boost** survive the very hard cuts on  $p_T^\tau$  (65-100 GeV) and  $p_T^{\text{miss}}$  (75-145 GeV)

**Consequence: Small opening angle** between the decay products  $\tau$  and  $\nu$

- **Signal:** Flat distribution (low mass), peak at  $\Delta\phi = \pi$  (high mass)

- **Efficiencies: Background:** 1-2%, **Signal:** 50-97%



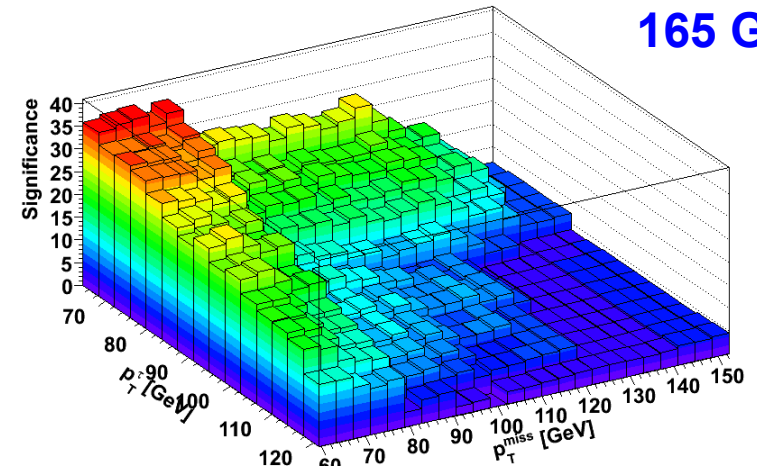
# $p_T$ -Scans

## Justification for three cut values sets

Keeping all other cut values constant, a scan of  $p_T^\tau / p_T^{\text{miss}}$  cut values was performed

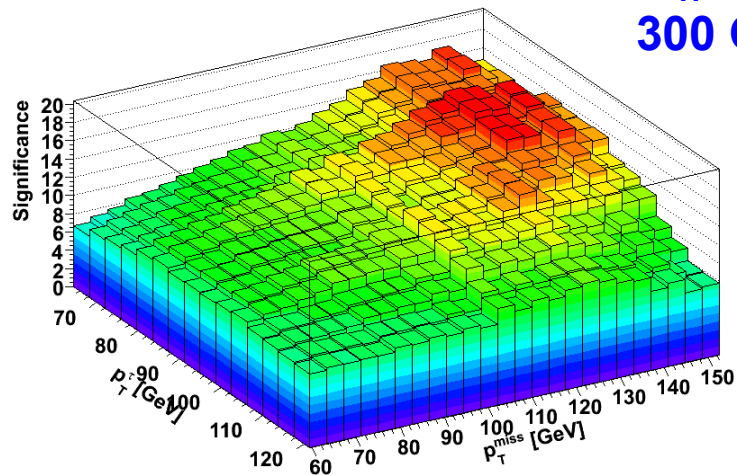
Scan of Transverse Momentum Cut Values for  $m_{H^\pm} = 165$  GeV

$m_{H^\pm} =$   
165 GeV



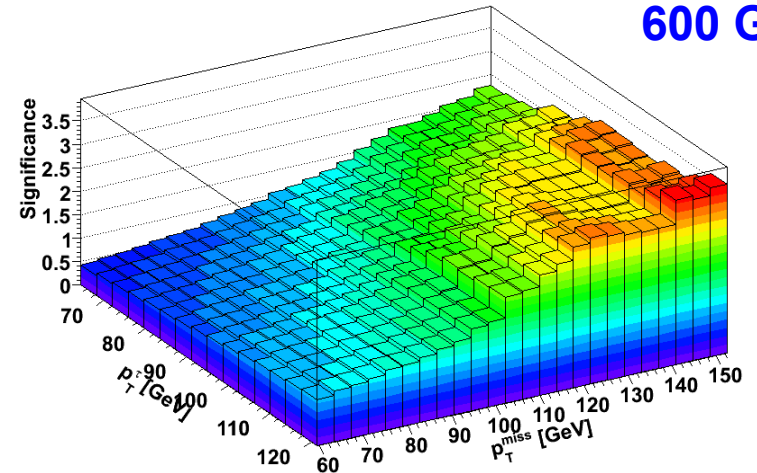
Scan of Transverse Momentum Cut Values for  $m_{H^\pm} = 300$  GeV

$m_{H^\pm} =$   
300 GeV



Scan of Transverse Momentum Cut Values for  $m_{H^\pm} = 600$  GeV

$m_{H^\pm} =$   
600 GeV

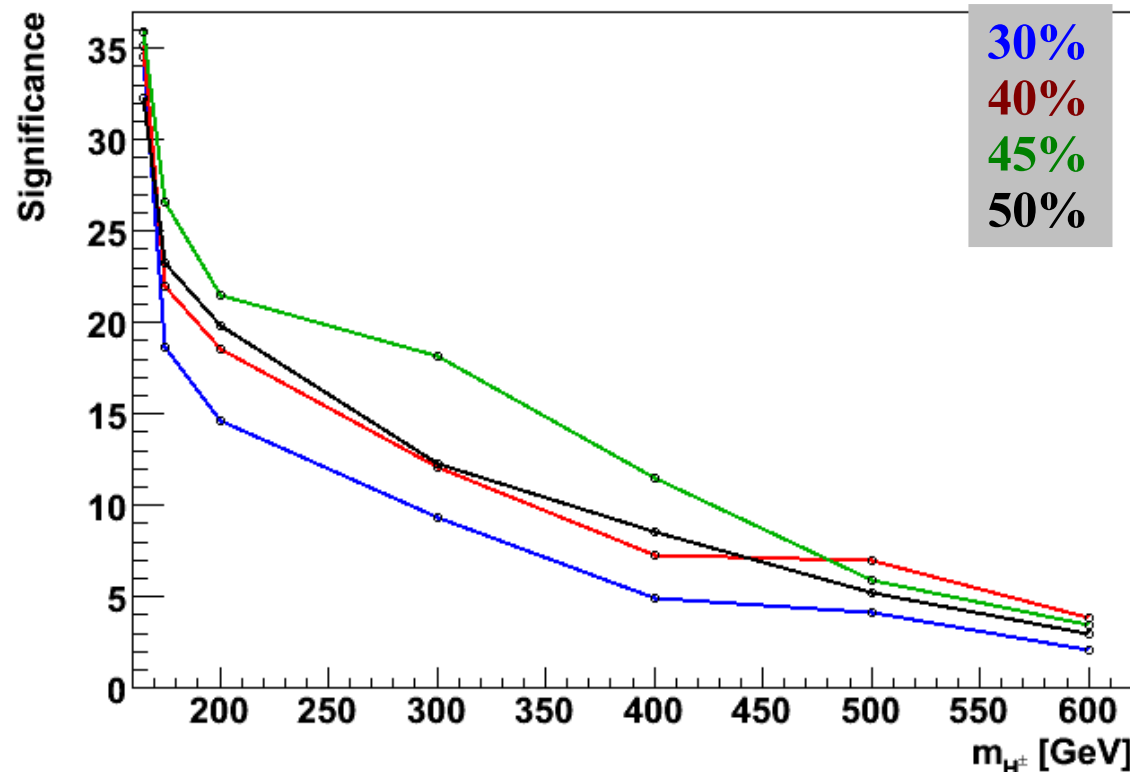


- $m_{H^\pm}$  165 GeV: Cut values for  $p_T^\tau > 80$  GeV or  $p_T^{\text{miss}} > 80$  GeV degrade results
- $m_{H^\pm}$  600 GeV: Same for  $p_T^\tau < 100$  GeV or  $p_T^{\text{miss}} < 120$  GeV



# Tau-Tagging

Significance: Comparison for different  $\tau$  efficiencies

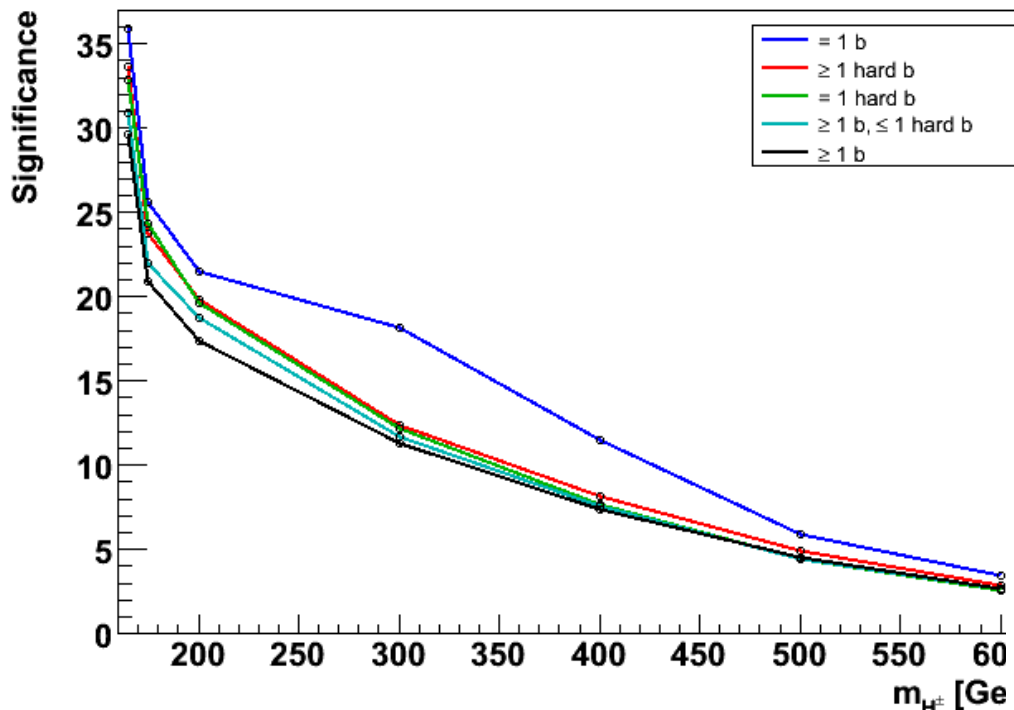


- Different  **$\tau$ -tagging efficiencies** were studied (with corresponding rejection factors)
- **Optimum: 45%** - optimizes **significance** for  $m_{H^\pm} < 450$  GeV
  - $m_{H^\pm} > 450$  GeV: close to optimum; increases **signal efficiency**



# b-tagging

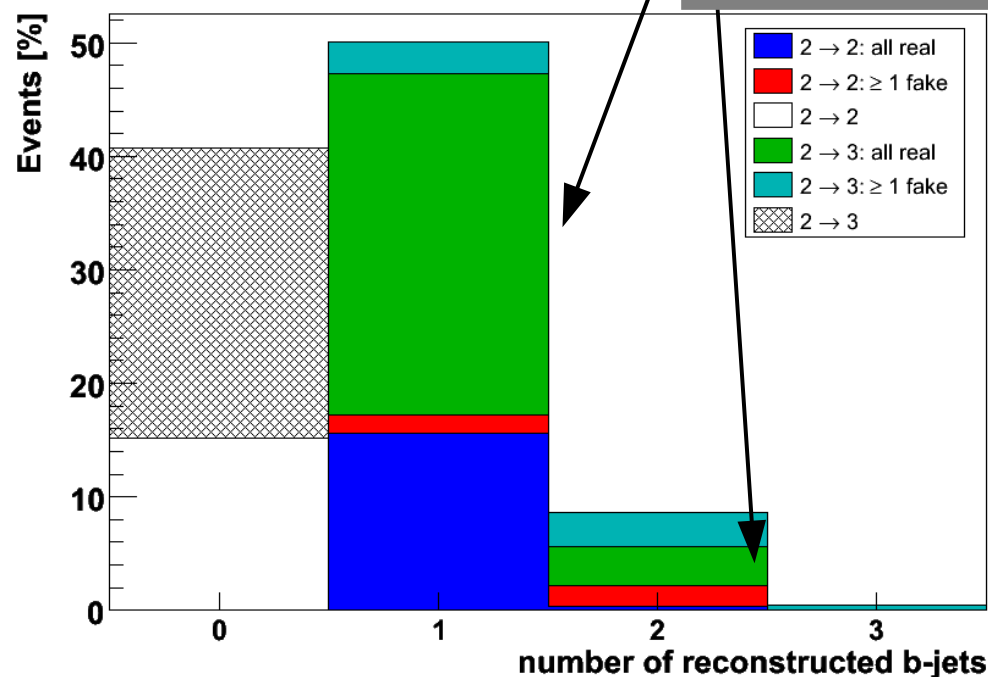
Significance as a function of  $m_{H^\pm}$  for different b-cuts



- Five b-tagging requirements were studied
- Optimum: requiring exactly 1 b-tag

- only in **10% of the events**  
**2 b-jets** reconstructed
- also for **165 GeV** (dominance of the  $2 \rightarrow 3$  process  $gg \rightarrow tbH^\pm$ ):

Reconstructed b-jets per process



$m_{H^\pm} = 165 \text{ GeV}$

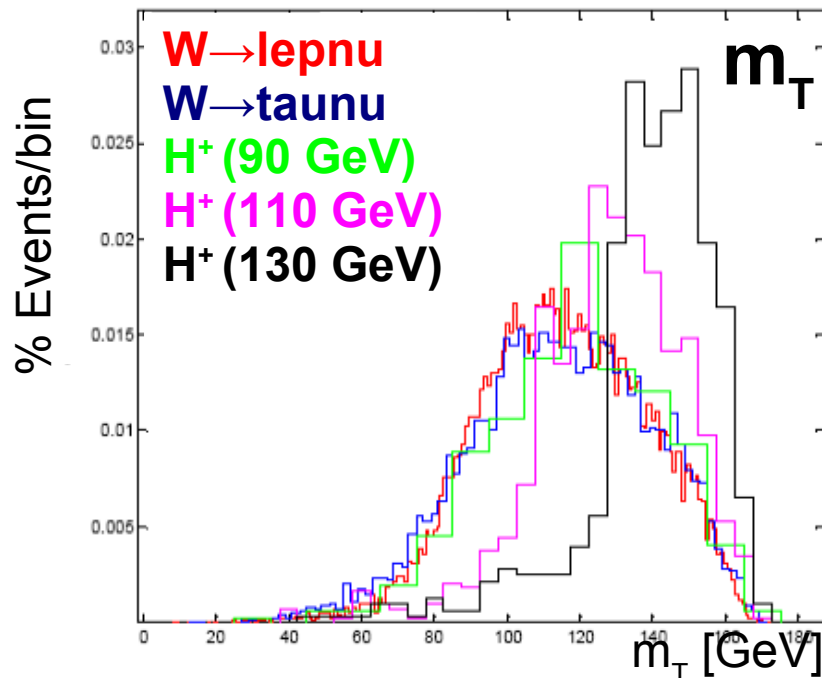
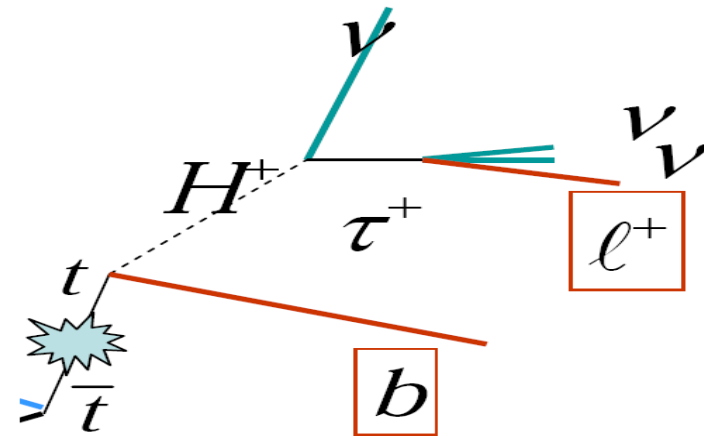
number of reconstructed b-jets



# Novel “Transverse Mass” Method

Ofer Vitells, Eilam Gross (Weizmann)

- a novel technique to reconstruct the **transverse mass in presence of the 3 neutrinos**, developed for this analysis [6]
- similar to  $W \rightarrow l\nu$  reconstruction



**use:**

$$[ p_{\parallel}^{\text{miss}} = (E, p_z) ]$$

$$(1) (M^{H^+})^2 = (P^{\text{miss}} + P^{\text{lep}})^2 = (p_{\parallel}^{\text{miss}} + p_{\parallel}^{\text{lep}})^2 - (p_T^{\text{miss}} + p_T^{\text{lep}})^2$$

$$(2) (M^{\text{top}})^2 = (P^{\text{miss}} + P^{\text{lep}} + P^b)^2$$

**maximize  $M_{H^+}$  wrt  $p_{\parallel}^{\text{miss}}$ , using  $M_{\text{top}}$  constraint (2)**

$$\frac{\partial}{\partial p_{\parallel}^{\text{miss}}} ((p_{\parallel}^{\text{miss}} + p_{\parallel}^{\text{lep}})^2 - \lambda (p_{\parallel}^{\text{miss}} + p_{\parallel}^{\text{lep}} + p_{\parallel}^b)^2) = 0$$

$$\Rightarrow (M_T^{H^+})^2 = (\sqrt{(M^{\text{top}})^2 + (p_T^{\text{lep}} + p_T^b + p_T^{\text{miss}})^2} - p_T^b)^2 - (p_T^{\text{miss}} + p_T^{\text{lep}})^2$$

**this value is confined to  $M^{H^+} \leq M_T^{H^+} \leq M^{\text{top}}$**