What if Majorana masses of right-handed neutrinos are below the electroweak scale?

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Minimal model with right-handed neutrinos:

$$\mathcal{L} = \mathcal{L}_{\text{MSM}} + \frac{1}{2}\bar{\tilde{N}}_s[i\partial\!\!\!/ - M_s]\tilde{N}_s - [h_{\alpha s}\bar{L}_\alpha \tilde{\phi}\, P_R \tilde{N}_s + \text{H.c.}] \ ,$$

where “active” generations $\alpha = 1, 2, 3$ and “sterile” generations $s = 1, 2, 3$ are summed over.

Active neutrino masses with the see-saw:

$$m_{\nu_\alpha} \approx \frac{|h_{\alpha s}|^2 v^2}{M_s} \sim \mathcal{O}(0.01 \text{ eV}) \ .$$

Mixing angle between active and sterile neutrinos:

$$\theta_{\alpha s} \equiv \frac{h_{\alpha s} v}{M_s} \ .$$

How to choose the new parameters M_s, $h_{\alpha s}$?

Usually it is assumed, motivated by the MSM where $10^{-6} \lesssim |h_{\alpha\beta}| \lesssim 1.0$, that $|h_{\alpha s}| \lesssim 1.0$.

In particular, choosing values not far from the upper bound, and matching to $\Delta m^2_{\mathrm{atm}}, \Delta m^2_{\mathrm{sol}}$, suggests

$$M_{1,2,3} \sim 10^{10}...10^{15}\ \text{GeV}\ .$$

This seems to offer a window to GUT physics...

.. and allows to explain B-asymmetry through decays of N_1, followed by anomalous conversion of L to B.

Fukugita, Yanagida 1986

However, from the point of view of the active neutrino masses, nothing prevents us from simultaneously decreasing M_s and $h_{\alpha s}$:

$$m_{\nu_\alpha} \approx \frac{|h_{\alpha s}|^2 v^2}{M_s} \sim \mathcal{O}(0.01\ \text{eV}) \ .$$

Let us explore whether this would lead to any interesting consequences for cosmology.

In particular, "νMSM":

Asaka, Blanchet, Shaposhnikov 2005

$M_1 \sim$ keV, $|h_{\alpha 1}| \lesssim 10^{-11}$

$\Rightarrow$ Dark Matter

$\Rightarrow \frac{|h_{\alpha 1}|^2 v^2}{M_1} \ll \Delta m_{\text{atm}}, \Delta m_{\text{sol}}$.

$M_{2,3} \sim$ GeV, $|h_{\alpha 2,3}| \lesssim 10^{-7}$

$\Rightarrow$ Baryogenesis

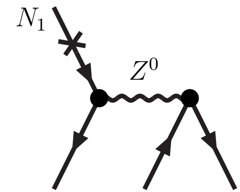
$\Rightarrow \Delta m_{\text{atm}}, \Delta m_{\text{sol}}$.

Mixing angles: $\theta_{\alpha s} \lesssim 10^{-4} \Rightarrow$ interactions **very** weak.

Cosmology of N_1

1. Decay width:

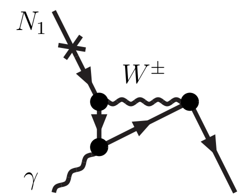
$$\Gamma_{N_1 \to \nu\nu\bar{\nu}} = \frac{G_F^2 M_1^5}{96\pi^3} \sum_{\alpha=e,\mu,\tau} |\theta_{\alpha 1}|^2 \ll H_0 \, .$$



   Could be dark matter, if produced!

2. Decay width to photons:

$$\Gamma_{N_1 \to \nu\gamma} = \frac{9\alpha_{\text{em}} G_F^2 M_1^5}{256\pi^4} \sum_{\alpha=e,\mu,\tau} |\theta_{\alpha 1}|^2 \, .$$

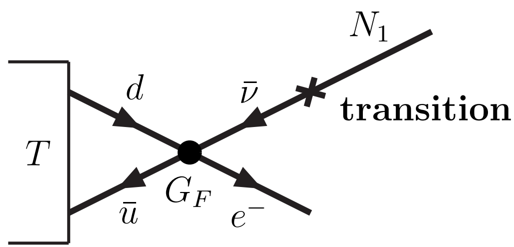


   Could be detected as an X-ray peak!

3. Free-streaming length is large: watch out that it does not erase small-scale structures!

Some N_1 are certainly produced thermally

Production peaks at $T \sim 200$ MeV $\ll m_W$, so one can understand it within the Fermi model.



Dodelson, Widrow 1994;
Dolgov, Hansen 2000;
Abazajian, Fuller, Patel 2001

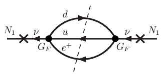
After their production N_1 do not interact any more.
[Their density is much below the equilibrium value.]

Careful theory computation of the abundance

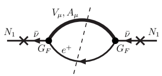
Asaka, Laine, Shaposhnikov 2006

Rate $\propto$ |amplitude|^2

$\Rightarrow$ the rate can be related to the imaginary part of the 2-point function of active neutrinos:



Or, more generally, when $d, \bar u$ not perturbative:



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As formulae

Cosmology part:

$$-T\frac{\mathrm{d}}{\mathrm{d}T}\left[\frac{n_1(T)}{s(T)}\right] = \frac{1}{3c_s^2(T)s(T)}\sqrt{\frac{3m_{\rm Pl}^2}{8\pi e(T)}}\int \mathrm{d}^3\mathbf{q}\, R(T,\mathbf{q}) \; .$$

Relation to active neutrinos:

$$R(T,\mathbf{q}) \sim \frac{1}{(2\pi)^3 2|\mathbf{q}|}\sum_{\alpha=1}^{3}\theta_{\alpha 1}^2 \frac{M_1^4\, \mathrm{Tr}[\not{Q}\, \mathrm{Im}\, \not{\Sigma}_\alpha]}{[M_1^2 + 2|\mathbf{q}|\, \mathrm{Re}\, \Sigma_\alpha]^2} \; .$$

Needed: $\mathrm{Re}\,\Sigma_\alpha$ = real part of active neutrino self-energy;
$\mathrm{Im}\,\Sigma_\alpha$ = imaginary part of active neutrino self-energy;
$e(T)$ = total energy density of the Universe;
$s(T)$ = total entropy density of the Universe;
$c_s^2(T)$ = speed of sound squared.

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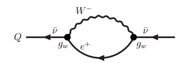
Real part of the active neutrino self-energy



$$\mathrm{Re}\,\Sigma_\alpha = Q\, a_\alpha(Q) + u\, b_\alpha(Q) \; , \quad u = (1, \mathbf{0})$$

$$b_\alpha(Q) = \frac{16 G_F^2 T^4}{\pi \alpha_w} q^0 \left[2\phi\left(\frac{m_{l_\alpha}}{T}\right) + \cos^2\theta_{\rm W}\, \phi\left(\frac{m_{\nu_\alpha}}{T}\right)\right] \; .$$

Nötzold, Raffelt 1988

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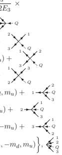
Imaginary part of the active neutrino self-energy

Perturbatively, the 2-loop graph gives:

$$\mathrm{Im}\,\not{\Sigma}_e^{\rm had}(Q) = 4N_{\rm c}G_F^2|V_{ud}|^2 n_{\rm F}^{-1}(q^0)\int\frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3 2E_1}\int\frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3 2E_2}\int\frac{\mathrm{d}^3\mathbf{p}_3}{(2\pi)^3 2E_3}\times$$
$$\times\Big\{ (2\pi)^4\delta^{(4)}(P_1+P_2+P_3-Q)\, n_{\rm F1}n_{\rm F2}n_{\rm F3}\,\mathcal{A}(-m_e,m_d,-m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(P_2+P_3-P_1-Q)\, n_{\rm F2}n_{\rm F3}(1-n_{\rm F1})\,\mathcal{A}(m_e,m_d,-m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(P_1+P_3-P_2-Q)\, n_{\rm F1}n_{\rm F3}(1-n_{\rm F2})\,\mathcal{A}(-m_e,-m_d,-m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(P_1+P_2-P_3-Q)\, n_{\rm F1}n_{\rm F2}(1-n_{\rm F3})\,\mathcal{A}(-m_e,m_d,m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(P_1-P_2-P_3-Q)\, n_{\rm F1}(1-n_{\rm F2})(1-n_{\rm F3})\,\mathcal{A}(-m_e,-m_d,m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(P_2-P_1-P_3-Q)\, n_{\rm F2}(1-n_{\rm F1})(1-n_{\rm F3})\,\mathcal{A}(m_e,m_d,m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(P_3-P_1-P_2-Q)\, n_{\rm F3}(1-n_{\rm F1})(1-n_{\rm F2})\,\mathcal{A}(m_e,-m_d,-m_u) +$$
$$+ (2\pi)^4\delta^{(4)}(-P_1-P_2-P_3-Q)\,(1-n_{\rm F1})(1-n_{\rm F2})(1-n_{\rm F3})\,\mathcal{A}(m_e,-m_d,m_u)\Big\} ,$$



where

$$\mathcal{A}(m_e,m_d,m_u) \equiv \gamma^\mu(\not{P}_1+m_e)\gamma^\nu\,\mathrm{Tr}\left[(\not{P}_2+m_d)\gamma_\mu a_L(\not{P}_3+m_u)\gamma_\nu a_L\right] .$$

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More generally:

$$\mathrm{Im}\,\not{\Sigma}_\alpha(Q) \sim G_F^2\int\frac{\mathrm{d}^3\mathbf{r}}{(2\pi)^3}\,\mathcal{K}\Big(\frac{|\mathbf{q}|}{T},\frac{|\mathbf{q}+\mathbf{r}|}{T}\Big)$$
$$\times\; \gamma^\mu(\not{Q}+\not{R})\gamma^\nu\,\rho_{\mu\nu}^{V,A}(|\mathbf{q}+\mathbf{r}|-|\mathbf{q}|,\mathbf{r}) \; ,$$

where $\mathcal{K}$ is a known thermal "kernel", and $\rho_{\mu\nu}^{V,A}$ represent flavour-singlet and non-singlet vector and axial current spectral functions ($\rho = \mathrm{Im}\,\Pi_R$).

The spectral functions get computable leptonic contributions, but also hadronic ones, which are hard to compute reliably in the temperature range of interest.

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Why is $T \sim 200$ MeV the relevant scale?

$\mathrm{Tr}[\not{Q}\,\mathrm{Im}\,\not{\Sigma}_\alpha] \sim G_F^2 T^6 f(|\mathbf{q}|/T)$.

$\mathrm{Re}\,\Sigma_\alpha \sim G_F^2 T^4 \alpha_w^{-1}|\mathbf{q}|$.

For $|\mathbf{q}| \sim T$, one then has

$$R(T,\mathbf{q}) \sim \frac{M_1^4 G_F^2 T^6}{(M_1^2 + 100 G_F^2 T^6)^2} \; ,$$

and the rate is strongly peaked around

$$T \sim \left(\frac{M_1}{10\, G_F}\right)^{\frac13} \sim 200\ \text{MeV}\left(\frac{M_1}{1\ \text{keV}}\right)^{\frac13} \; .$$

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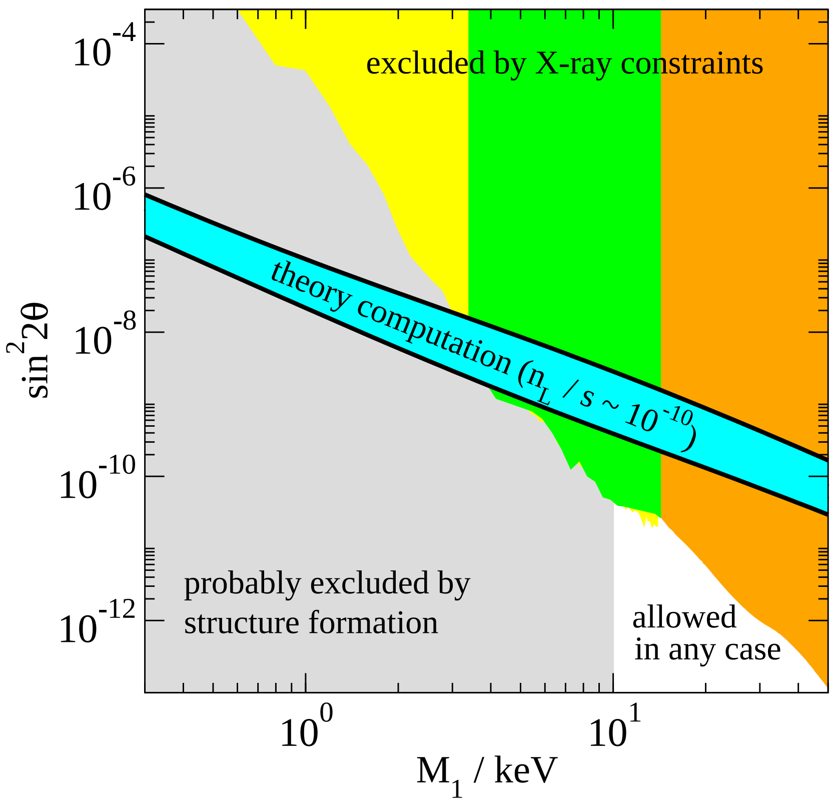
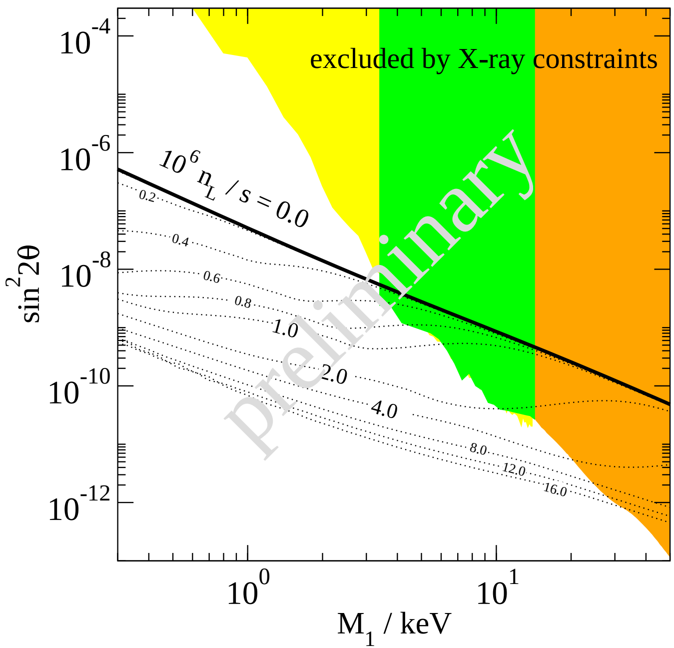
After the computation, can compare the theoretical result with experimental constraints.



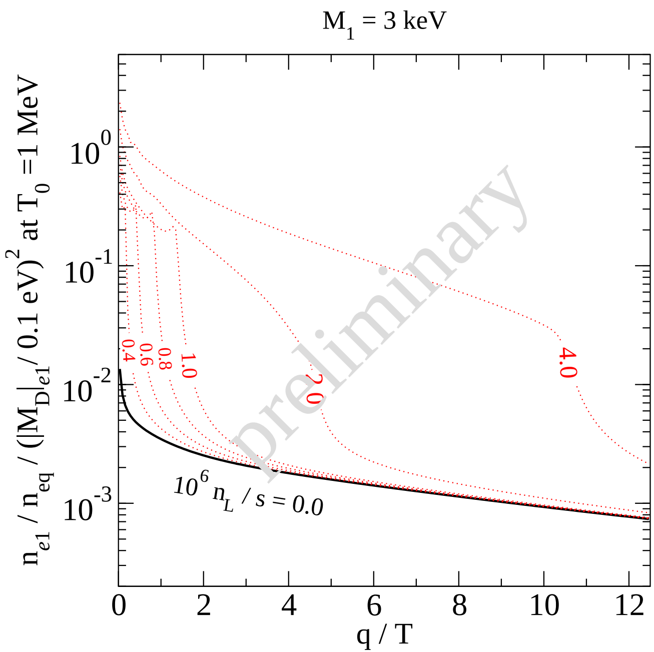
The situation changes dramatically if $n_L/s \gg 10^{-10}$, because of an MSW-related resonance.

Shi, Fuller 1998



Asaka, Laine, Shaposhnikov 2008

Note also that the distribution function of sterile neutrinos is very much out-of-equilibrium:



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So structure formation constraints may change.

Conclusions

Light right-handed neutrinos may be even more useful for cosmology than heavy right-handed neutrinos.

There are many parameters, but also many constraints, so that we can focus on fairly definite scenarios now.

Predictions

Ultimately a peak around $E \approx M_1$ should be detected in the X-ray background.

Absolute mass scale of active neutrinos is given by $\Delta m_{\mathrm{atm}} \sim 0.05$ eV, so that $m_{\nu_\alpha} \leq \Delta m_{\mathrm{atm}}$.