Plaquette expectation value and lattice free energy of three-dimensional SU(N_c) gauge theory

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GRASPANP-HANUC Lecture Week in Hyytiälä on September 25-30, 2006

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[hep-lat/0609015]

Introduction

- Weak coupling expansion of free energy in hot QCD:

$$\frac{f}{f_{SB}} = 1 + g^2 + g^3 + g^4 \ln g + g^4 + g^5 + g^6 \ln g + g^6 + \ldots$$

- Due to infrared divergences, g^6 gets contributions from infinite number of diagrams.
  $\longrightarrow$ g^6 is non-perturbative, *needs numerical evaluation*
- Direct numerical evaluation *very* demanding (150Tflop/s).
  - quarks
  - high T

Dimensional reduction

- At high temperatures ($T \gtrsim 2T_c$) all the non-perturbative physics can be isolated into a three-dimensional, super-renormalizable pure gauge theory, MQCD.

$$S^{\mathrm{MQCD}} = \int \mathrm{d}^3x \frac{1}{2g_3^2} \sum_{k,l=1}^{3} \mathrm{Tr}[F_{kl}^2]$$

$$= \beta \sum_x \sum_{k<l}^{3} \left(1 - \frac{1}{N_c} \mathrm{ReTr}[P_{kl}(x)]\right), \text{with } \boxed{\beta = \frac{2N_\mathrm{c}}{g_3^2 a}}$$

The quantity we are after is the free energy density:

$$f_{\overline{\mathrm{MS}}} = -g_3^6 \frac{d_A N_\mathrm{c}^3}{(4\pi)^4} \left[\left(\frac{43}{12} - \frac{157}{768}\pi^2\right) \ln \frac{\bar{\mu}}{2N_\mathrm{c} g_3^2} + B_\mathrm{G}(N_\mathrm{c})\right].$$

N_c-dependence

Study N_c-dependence of free energy in effective theory because:

- it acts as a consistency check for dimensional reduction.
- Large-N_c limit simplifies but is similar in phenomenology. Planar diagram theory, Eguchi-Kawai model ...
- Strings, GUTs

Lattice computations

Outline of lattice calculations:

$$B_{\mathrm{G}}(N_{\mathrm{c}}) - \left(\frac{43}{12} - \frac{157}{768}\pi^2\right) c_4' = P_{\mathrm{G}}(\infty, N_{\mathrm{c}}).$$

$$P_{\mathrm{G}}(\beta, N_{\mathrm{c}}) \equiv \frac{32\pi^4\beta^4}{d_A N_{\mathrm{c}}^6}\left\{\langle 1 - \frac{1}{N_{\mathrm{c}}}\mathrm{Tr}[P]\rangle_a - \left[\frac{c_1}{\beta} + \frac{c_2}{\beta^2} + \frac{c_3}{\beta^3} + \frac{c_4}{\beta^4}\ln\beta\right]\right\}$$

- Measure $\langle 1 - \frac{1}{N_c}\mathrm{Tr}[P_{12}]\rangle$ as a function of β for $N_{\mathrm{c}} = 2, 3, 4, 5, 8$
- Subtract ultraviolet divergences.
- Deal with finite lattice.
- Extrapolate $\beta \to \infty$
- Find the functional form of non-perturbative input $P_{\mathrm{G}}(N_{\mathrm{c}}, \infty)$.

Significance loss

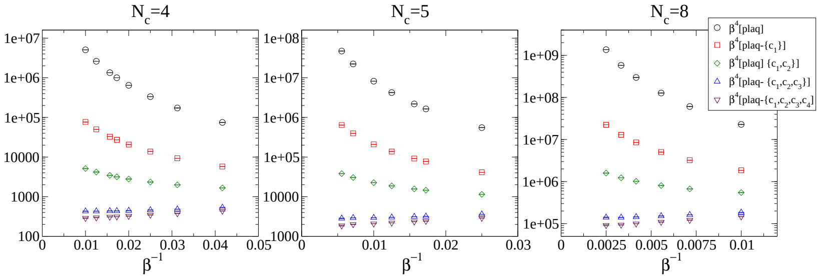


Figure: Massive significance loss due to the subtraction of ultraviolet divergences in the plaquette expectation value. The physics is in the sixth decimal of the plaquette.

Finite lattice

There is only one scale in the theory the "glue-ball" correlation length $\sim 1/N_c g_3^2$, which is also the confinement scale.

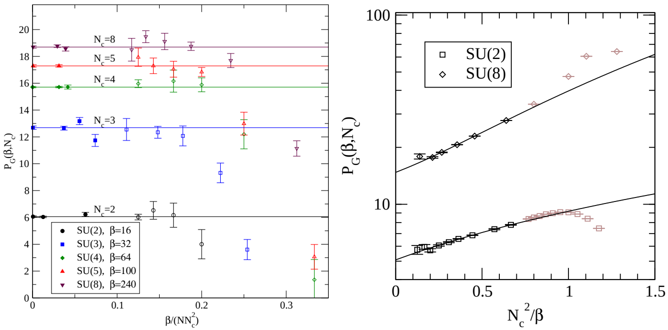


Figure: $P_{\rm G}(\beta, N_c)$ as a function of the physical lattice size and physical lattice spacing. $\longrightarrow N_c^2 < \beta \lesssim N(N_c/3)^2$

Continuum limit

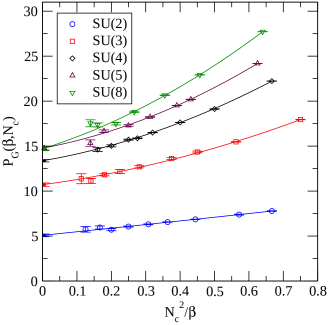
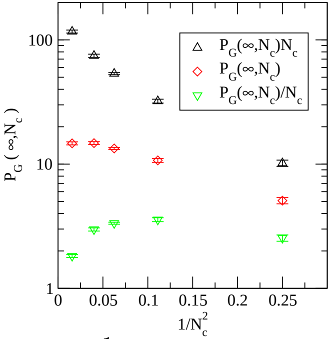
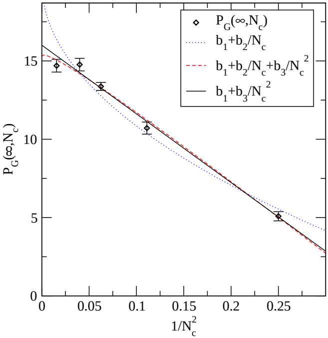


Figure: Continuum extrapolations of $P_{\rm G}(\beta, N_{\rm c})$ for each $N_{\rm c}$.

- We expect systematical errors of order 1-σ from the choice of the fitting function.

N_c-dependence





- Term N_c^{-1} gives a very bad description of data or has coefficient consistent with zero within our resolution.

$$P_G(N_c,\infty) = 15.9(2) - 44(2)N_c^{-2} \qquad (= 11.0 \pm 0.3, \text{ for } N_c = 3)$$

conclusions

- At high temperatures ($T \gtrsim 2T_c$) non-perturbative physics of hot QCD can be isolated into a simpler 3d pure gauge theory.
- We have studied the $N_{\rm c}$-dependence of plaquette expectation value and free energy of the effective theory.
  - High precision lattice measurements of plaquette expectation value with $N_{\rm c} = 2, 3, 4, 5, 8$ were performed[1].
  - Non-perturbative input $P_{\rm G} = 15.9(2) - 44(2)N_{\rm c}^{-2}$, seems to be a function $N_{\rm c}^2$.
  - Higher order terms are small and physical case $N_{\rm c} = 3$ is well described by this form.

[1]Total of 1.2×10^{17} flops were used.