Seminar 5

26. helmikuuta 2008
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diagrammatic technique

1) δN - formalism

ζ is conserved at superhubble scales

$(1+w)\dot{\zeta} = O(\nabla\varphi) \Rightarrow \zeta = \text{Const}$ for $k < aH$.

$ds^2 = dt^2 - a^2(t)\, e^{2\zeta(x)} d\vec{x}^2$

$\tilde{a} = a e^{\zeta}$

$\delta N = \ln \frac{\tilde{a}}{a} = \zeta$

N depends on various light fields:

$\zeta = N_A \delta\varphi^A + \frac{1}{2} N_{AB} \delta\varphi^A \delta\varphi^B + \dots$

One field:

$\zeta = N' \delta\varphi + \frac{1}{2} N'' (\delta\varphi)^2 + \dots$ - inflaton

$\langle \delta\varphi^2 \rangle = \frac{H_*}{(2\pi)^2}$ - gaussian field

Moreover,

$\langle \delta\varphi^A \delta\varphi^B \rangle = G(|x_1 - x_2|)\, \delta^{AB}$

2) How to calculate multipoint corr. functions of ζ?

2.1) Real space

$$\langle \varphi^A_{x_1} \varphi^B_{x_2} \rangle = \begin{cases} G(|x_1 - x_2|)\, \delta^{AB}, & x_1 \neq x_2 \\ \langle \varphi^2 \rangle\, \delta^{AB}, & x_1 = x_2 \end{cases}$$

a) draw n points $x_1, x_2, x_3, \dots, x_n$; connect them with r propagators, $r > n - 1$

a*) assign factor $N_{AB\dots}$ to each vertex

b) Label end points of propagator $A, B, \dots$

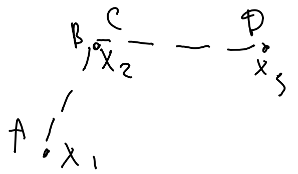
c) Assign a factor of $\delta^{AB} G(|x_1 - x_2|)$ to each propagator

d) When l propagators connect the same x_i and x_j this gives $l!$. Dressing gives 2^l.

e) Allow all permutations of points x_i

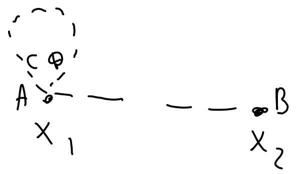
Examples:

a) tree level 3-point function



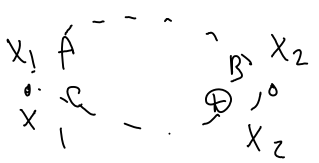
$$N_A N_{BC} N_D \, G(x_1 - x_2) \, G(x_2 - x_3) \times$$
$$\times \delta_{AB} \delta_{CD} + 2 \text{ perm.} =$$
$$= N_B N_{BD} N_D \, G G + 2 \text{ perm.}$$

b) Loop corrections to $\langle \zeta_1 \zeta_2 \rangle$



$$= \frac{1}{2} N_{ACD} N_B \langle \varphi^2 \rangle \delta_{CD} \delta_{AB} \, G(x_1 - x_2)$$

$=$ same



$$= \frac{1}{2} N_{AC} N_{BD} \, G(x_1 - x_2) \, G(x_1 - x_2) \delta_{AB} \delta_{CD}$$

2.2) Momentum space

1) n external solid lines, r propagators. Every vertex contains 1 solid line and some number of propagators attached

2) Label external lines with momenta $k_1, k_2, \ldots$ Label propagators with momenta q_i, their ends – with $A, B, C, \ldots$

3) Assign factor $N_{\ldots}$ $(2\pi)^3 \delta(k_i - q_1 - \ldots - q_r)$

3) Assign factor $N_{AB,\dots}(2\pi)^3\,\delta(k_i - q_1 - \dots - q_n)$ to each vertex.
Number of derivatives $N_{AB\dots}$ = number of propagators attached

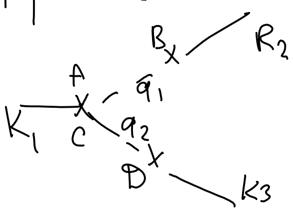
4) Attach factor $\delta^{AB} P(q)$ to each prop.

5) integrate over all momenta q_i
$\int \frac{dq_i}{(2\pi)^3}$

6) Numerical factor: ℓ propagators end with the same vertices — factor $\ell!$
Dressed vertex — 2^ℓ

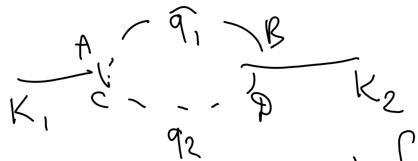
Examples:

1) tree level 3-point function



$$\int \frac{dq_1}{(2\pi)^3}\frac{dq_2}{(2\pi)^3} N_{AC} N_B N_D\, \delta_{AB}\delta_{CD}\times$$
$$\times (2\pi)^9\, \delta(k_1 - q_1 - q_2)\,\delta(k_2 - q_1)\times$$
$$\times\, \delta(k_3 - q_2)\, P(q_1) P(q_2) =$$
$$= N_{AD} N_A N_D\, (2\pi)^3 \delta(k_1 - k_2 - k_3)\, P(k_2) P(k_3) + 2\,\text{perms}$$

2) Loops: power spectrum



$$= \frac{1}{2} N_{AC} N_{BD}\, \delta_{AB}\delta_{CD}\times$$
$$\times \int \frac{1}{(2\pi)^6} dq_1 dq_2\, (2\pi)^6 \delta(k_1 - q_1 - q_2)$$
$$\times\, \delta(q_1 + q_2 - k_2) \times P(q_1) P(q_2) =$$
$$= \frac{1}{2} N_{AB} N_{AB}\, \delta(k_1 - k_2) \int dq\, P(q)\, P(k_1 - q)$$