Quantum information and computing 2007: exercise from lecture 12

Jani-Petri.Martikainen@helsinki.fi

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Note: Return before 10:00 on wednesday 2.5

1. Suppose we have a single qubit principal system, interacting with a single qubit environment through
$$U = \frac{X}{\sqrt{2}} \otimes I + \frac{Y}{\sqrt{2}} \otimes X. \tag{1}$$
Give the quantum operation for this process, in the operator-sum representation. (First operators act on the system and the environment starts in the state $|0\rangle$.)

2. What is the fidelity between the density operators
$$\frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|; \quad \frac{2}{3}|0\rangle\langle 0| + \frac{1}{3}|1\rangle\langle 1| \tag{2}$$
Between:
$$\frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|; \quad \frac{2}{3}|+\rangle\langle +| + \frac{1}{3}|-\rangle\langle -| \tag{3}$$
(Recall that $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$.)

3. **Von Neuman entropy** of a quantum state ρ is defined by
$$S(\rho) = -Tr\left(\rho \log \rho\right). \tag{4}$$
Calculate $S(\rho)$ for states
$$\rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \rho = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad \rho = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \tag{5}$$