Some simple examples with conditional Wiener measure

$$\int_{C\{x_0,0;\,x,t\}} F[x(\tau)]\, d_W x(\tau)$$

Note that

$$\int_{C\{0,0;\,t\}} F[x(\tau)]\, d_W x(\tau) = \int_{-\infty}^{\infty} dx \int_{C\{0,0;\,x,t\}} F[x(\tau)]\, d_W x(\tau)$$

Example 1'

$$I = \int_{C\{0,0;x,t\}} x(t_e)\, d_W x(\tau)$$

Clearly,

$$I = \int_{-\infty}^{\infty} \frac{e^{-\frac{x_e^2}{t_e}}}{\sqrt{\pi t_e}}\, x_e\, \frac{e^{-\frac{(x-x_e)^2}{t-t_e}}}{\sqrt{\pi(t-t_e)}}\, dx_e$$

$$\frac{x_e^2}{t_e} + \frac{x^2-2xx_e+x_e^2}{t-t_e} = \frac{(t-t_e)x_e^2 + t_e x^2 - 2t_e x x_e + t_e x_e^2}{t_e(t-t_e)}$$

$$= \frac{t x_e^2 + t_e x^2 - 2t_e x x_e}{t_e(t-t_e)} = \frac{t}{t_e(t-t_e)}\left[\left(x_e^2 - 2x_e \frac{t_e}{t}x + \left(\frac{t_e}{t}x\right)^2\right) - \left(\frac{t_e}{t}x\right)^2 + \frac{t_e}{t}x^2\right]$$

$$= \frac{t}{t_e(t-t_e)}\left(x_e - \frac{t_e}{t}x\right)^2 + \frac{x^2}{t}$$

$$I = \int_{-\infty}^{\infty} dx_e \frac{1}{\pi\sqrt{t_e(t-t_e)}}\, e^{-\frac{x^2}{t}}\, e^{-\frac{t}{t_e(t-t_e)}\left(x_e - \frac{t_e}{t}x\right)^2} \Big[\underbrace{\left(x_e - \frac{t_e}{t}x\right)}_{\int_{-\infty}^{\infty}\text{even}\cdot\text{odd}} \overset{\nearrow 0}{} + \frac{t_e}{t}x\Big]$$

$$= \frac{1}{\pi\sqrt{t_e(t-t_e)}}\, \frac{t_e}{t}x \sqrt{\frac{\pi t_e(t-t_e)}{t}}\, e^{-\frac{x^2}{t}} = \frac{x\, t_e}{t\sqrt{\pi t}}\, e^{-\frac{x^2}{t}}$$

Obviously, $\displaystyle\int_{-\infty}^{\infty} dx\, \frac{x\, t_e}{t\sqrt{\pi t}}\, e^{-\frac{x^2}{t}} = 0$ (in general $= x_0$)