Introduction to Path Integrals Homework 1
Spring 2011

1. Show that the Poisson distribution

$$P_n(t) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$

satisfies the semigroup property (i.e. Einstein-Smoluchowski-Kolmogorov-Chapman relation), where the convolution is defined as

$$\sum_{k=0}^{n} P_k(\tau)P_{n-k}(t-\tau) = \sum_{k=0}^{n} e^{-\lambda\tau}\frac{(\lambda\tau)^k}{k!}e^{-\lambda(t-\tau)}\frac{(\lambda(t-\tau))^{(n-k)}}{(n-k)!} \; .$$

Thus the Poisson distribution defines a Markovian stochastic process with the transition probability $W(n,t|n_0,t_0) \equiv P_{n-n_0}(t-t_0)$. Of course this is not Gaussian and, hence, not a Wiener process.

2. Calculate the transition probability $W(0,t|0,0)$ written in terms of the discrete approximation

$$\begin{aligned} W(0,t|0,0) &= \lim_{\epsilon\to 0}\lim_{N\to\infty}\frac{1}{(\sqrt{4\pi D\epsilon})^{N+1}}\int_{-\infty}^{\infty}dx_1\int_{-\infty}^{\infty}dx_2 ...\int_{-\infty}^{\infty}dx_N \\ &\times \; \exp\{-\frac{1}{4D\epsilon}\sum_{i=0}^{N}(x_{i+1}-x_i)^2\} \end{aligned}$$

performing the integrations over $x_1, x_2, ..., x_N$ one by one.

Hint: Each Gaussian integral for neighboring points satisfies the semigroup property.

3. Calculate the following Wiener integral (called triple correlation) with unconditional measure

$$I_2 = \int_{C\{x_0,0;t\}} x(\tau_1)x(\tau_2)x(\tau_3)d_W x(\tau) \; ,$$

where $\tau_1,\ \tau_2,\ \tau_3$ are three fixed moments of time satisfying $0 < \tau_1 < \tau_2 < \tau_3 < t$. Notice that the end point x is integrated over (in other words, it is not a conditional measure).

Hint: Follow precisely the way it was done for the two-point correlation function in Example 2 of Lecture Notes, p.13.

Please return the solutions into the box on the second floor of the Physicum building by Monday 31 January, 10:00. The first exercise session will be on Monday, 31 January at 14:15 in the room D116.