Advanced Space Physics (10 ECTS credits) Spring 2012

- Periods III & IV: 17.1. – 3.5.2012
- Lectures: Tue & Thu 12 – 14 (D117)
  - Hannu Koskinen (D 316; Hannu.E.Koskinen@helsinki.fi)
- Exercises: Mon 12 – 14 (BK106 Exactum!)
  - Heli Hietala (Heli.Hietala@helsinki.fi)

Some weeks lecture and exercise times will be interchanged
Information will be posted on the web-site

Course material

- Lectures
  - Text-book: Hannu E. J. Koskinen, Physics of Space Storms – From the Solar Surface to the Earth, Springer/PRAXIS 2011
    - focus on Chapters 2 – 11
    - can be downloaded from SpringerLink (google: Physics of Space Storms)
  - Additional material and exercise problems
    - http://theory.physics.helsinki.fi/~plasma_jatko/
    - Theoretical Physics → Lecturing Program → Advanced space physics
- Home exercises
  - Essential part of learning
  - Problems will often be discussed before the material be covered on the lectures!
  - Procedure during an exercise session:
    1. students present the solutions to problems of set n
    2. some of the students will be assigned to produce model solutions
    3. initiation of solving the problems of set $n + 1$

Supplementary reading

- Baumjohann, W., Treumann, R., Basic Space Plasma Physics, Imperial College Press, 1996.
- Bellan, P. M., Fundamentals of Plasma Physics, Gambridge University Press, 2006.
- Bittencourt, J. A., Fundamentals of Plasma Physics, Pergamon Press, 1986.
- Boyd,T. J. M., and Sanderson, J. J., The Physics of Plasmas, Cambridge University Press, 2003.
- Goldston, R. J., and Rutherford, P. H., Introduction to Plasma Physics, IOP Physics Publishing Ltd., 1995.
- Gurnett, D. A., and Bhattacharjee, A., Introduction to Plasma Physics. With Space and Laboratory Applications, Gambridge University Press, 2004.
- Krall, N. A., and Trivelpiece, A. W. , Principles of Plasma Physics, San Francisco Press, 1986 (a reproduction of the original text published by McGraw-Hill, 1973).
- Kulsrud, R. M., Plasma Physics for Astrophysics, Princeton University Press, 2005
- Nicholson, D. R., Introduction to Plasma Theory, John Wiley & Sons, 1983.
- Parks, G. K., Physics of Space Plasmas. An Introduction (2nd ed), Westview Press, 2004.
- Priest, E., and Forbes, T., Magnetic Reconnection, MHD Theory and Applications, Cambridge University Press, 2000.
- Sturrock, P. A. Plasma Physics, An Introduction to the Theory of Astrophysical, Geophysical & Laboratory Plasmas, Cambridge University Press, 1994.
- Treumann, R., and Baumjohann, W. Advanced Space Plasma Physics, Imperial College Press, 1997.

Preliminary plan of the lectures

- Introduction and physical foundations
- Single particle motion
- Waves in cold plasma approximation
- Vlasov theory
- Magnetohydrodynamics
- Space plasma instabilities
- Magnetic reconnection
- Plasma radiation and scattering
- Transport and diffusion in space plasmas
- Shocks and shock acceleration

Plasma state

Plasma is *quasi-neutral ionized gas* containing enough *free charges* to make *collective electromagnetic effects* important for its physical behaviour.

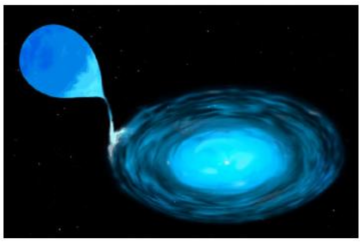
- ionization
  - 0.1% clear plasma properties
  - 1% almost perfect conductivity
- fourth state of matter: solid → liquid → gas → plasma
  - gradual, no phase transition
- production: heating, ionizing radiation, collisional ionization, electric discharges

Some 99.9…% of baryonic matter in the Universe is in plasma state

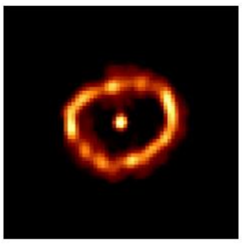
Dynamical natural plasmas near us

and a little further away

Plasma processes in
active galactic nuclei

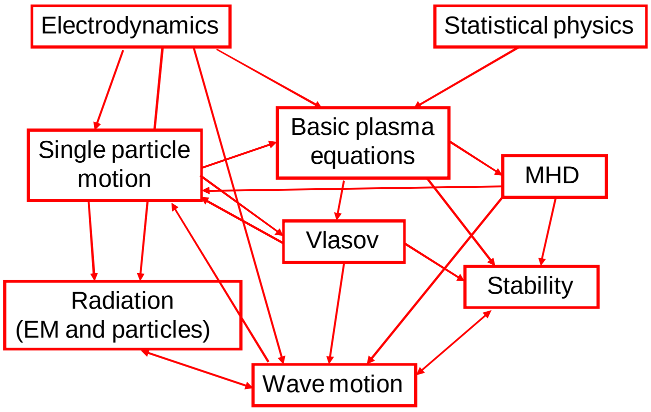


Accretion discs



Expanding shell of
Nova Cygni 1992 explosion

What kind of physics is involved

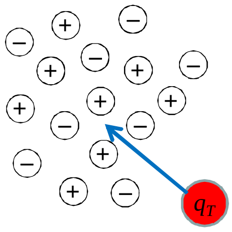


Consequently, plasma physics IS challenging

Prerequisites for this course

- Classical electrodynamics mechanics
- Thermodynamics, elements of (classical) statistical physics
- Mathematical methods for physics
  - vector analysis
  - complex analysis (incl. the idea of calculus of residues)
  - special functions
- Computer tools such as Mathematica, Maple, Matlab,… may sometimes be very useful
- Readiness to work hard!!

Debye screening



Coulomb potential of each charge: $\varphi = \dfrac{q}{4\pi\epsilon_0 r}$

Assume thermal equilibrium (Boltzmann distribution)

$$n_\alpha(\mathbf{r}) = n_{0\alpha}\exp\left(-\frac{q_\alpha \varphi}{k_B T_\alpha}\right)$$

α labels the particle populations (e.g., e, p)

Introduce a test charge q_T. What will be its potential?

Home exercise in introductory plasma physics: $\varphi = \dfrac{q_T}{4\pi\epsilon_0 r}\exp\left(-\dfrac{r}{\lambda_D}\right)$; $\lambda_D^{-2} = \dfrac{1}{\epsilon_0}\sum_\alpha \dfrac{n_{0\alpha}q_\alpha^2}{k_B T_\alpha}$

Debye length: $\lambda_D \propto \sqrt{\dfrac{T}{n}}$

Plasma parameter: $\Lambda = n_0 \lambda_D^3 \gg 1$

Number of particles in a Debye sphere: $N = \dfrac{4\pi}{3} n_0 \lambda_D^3$

"Definition of plasma"

$$\frac{1}{\sqrt[3]{n_0}} \ll \lambda_D \ll L$$

L is the size of the system

Plasma oscillation

Assume: n_0 fixed ions (+) & n_0 moving electrons (–)

Apply a small electric field $\mathbf{E}_1$

→ electrons move:

$$n_+ = n_0$$
$$n_- = n_0 + n_1(\mathbf{r},t) \quad ; \quad n_1 \ll n_0$$

Electron continuity equation: $\dfrac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0$

$n = n_0 + n_1(\mathbf{r},t)$; $\mathbf{u} = \underbrace{\cancel{\mathbf{u}_0}}_{0} + \mathbf{u}_1(\mathbf{r},t)$ ($\mathbf{u}_0 = 0$ ↔ electrons are assumed cold)

$$\underbrace{\cancel{\frac{\partial n_0}{\partial t}}}_{0} + \frac{\partial n_1}{\partial t} + \nabla\cdot((n_0+n_1)\mathbf{u}_1) = 0 \quad \Rightarrow \quad \frac{\partial n_1}{\partial t} + n_0\nabla\cdot\mathbf{u}_1 + \nabla\cdot(\underbrace{\cancel{n_1\mathbf{u}_1}}_{0\ \text{2nd order}}) = 0$$

Linearized continuity equation (1st order terms only): $\dfrac{\partial n_1}{\partial t} + n_0\nabla\cdot\mathbf{u}_1 = 0$

Force: $\mathbf{F} = q\mathbf{E} \Rightarrow m_e\dfrac{\partial \mathbf{u}_1}{\partial t} = -e\mathbf{E}_1$

1st Maxwell: $\nabla\cdot\mathbf{E}_1 = -en_1/\epsilon_0$

$$\Rightarrow \quad \frac{\partial^2 n_1}{\partial t^2} + \left(\frac{n_0 e^2}{\epsilon_0 m_e}\right) n_1 = 0$$

plasma frequency: $\omega_{pe}^2 = \dfrac{n_0 e^2}{\epsilon_0 m_e}$

Useful to remember

Plasma frequency (angular frequency) $\omega_{pe}^2 = \dfrac{n_0 e^2}{\epsilon_0 m_e}$

$$\boxed{f_{pe}(\mathrm{Hz}) = \frac{\omega_{pe}}{2\pi} \approx 9,0 \cdot \sqrt{n(\mathrm{m}^{-3})}}$$

Debye length $\boxed{\lambda_D(\mathrm{m}) \approx 7,4\sqrt{T(\mathrm{eV})/n(\mathrm{cm}^{-3})}}$

Note the units !
($1\ \mathrm{eV} \approx 1.16 \cdot 10^4\ \mathrm{K}$)

Gyromotion in the magnetic field

$$\mathbf{F} = q_\alpha(\mathbf{v} \times \mathbf{B}) \Rightarrow$$

$$\omega_{c\alpha} = \frac{|q_\alpha|B}{m_\alpha} \quad ; \quad f_{c\alpha} = \frac{\omega_{c\alpha}}{2\pi}$$

$$\boxed{\begin{aligned} f_{ce}(\mathrm{Hz}) &\approx 28 \cdot B(\mathrm{nT}) \\ f_{cp}(\mathrm{Hz}) &\approx 1.5 \cdot 10^{-2} \cdot B(\mathrm{nT}) \end{aligned}}$$

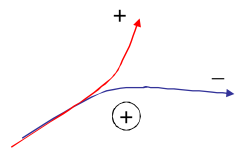
Collisions

Cross section: σ (m^2) Mean free path: $l_{mfp} = 1/(n\sigma)$

Collision frequency: $\nu_c = n\sigma v$

Weakly ionized plasmas

- charge ↔ neutral



Fully ionized plasmas

- Coulomb collisions
- small-angle collisions dominate, i.e., long-range force (exerc)

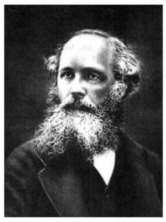
e.g. $\nu_{ei} = \dfrac{2n_0(Ze^2)^2 \ln\Lambda}{\epsilon_0^2 m_e^2 v_0^3} \propto \dfrac{\sqrt{n}\ln\Lambda}{\Lambda}$; $\ln\Lambda \simeq 10 - 30$ Coulomb logarithm

If $T \rightarrow$ large and/or $n \rightarrow$ small, then $\Lambda \rightarrow$ large ; $1/\Lambda \rightarrow$ small

Plasma becomes "collisionless" and the effects of Coulomb collisions are are included through average electromagnetic fields $\langle\mathbf{E}\rangle$ and $\langle\mathbf{B}\rangle$

Long-range small-angle collisions dominate

Maxwell's equations

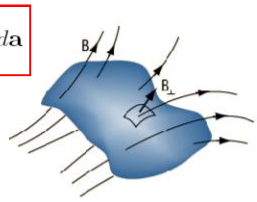

James Clerk Maxwell

$$\boxed{\begin{aligned}\nabla\cdot\mathbf{E} &= \rho/\epsilon_0\\ \nabla\cdot\mathbf{B} &= 0\\ \nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}\\ \nabla\times\mathbf{B} &= \mu_0\mathbf{J}+\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}\end{aligned}}$$

$$\begin{aligned}c &= 1/\sqrt{\epsilon_0\mu_0} = 299792458\ \mathrm{m/s}\\ \epsilon_0 &\approx 8,854\cdot 10^{-12}\ \mathrm{As/Vm},\\ \mu_0 &= 4\pi\cdot 10^{-7}\ \mathrm{Vs/Am},\end{aligned}$$

We call $\mathbf{B}$ magnetic field. It actually is the density of magnetic flux

$$\boxed{\Phi = \int_S \mathbf{B}\cdot d\mathbf{a}}$$



In electromagnetic media:

$$\begin{aligned}\nabla\cdot\mathbf{D} &= \rho_{free}\\ \nabla\cdot\mathbf{B} &= 0\\ \nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}\\ \nabla\times\mathbf{H} &= \mathbf{J}_{free}+\frac{\partial\mathbf{D}}{\partial t}\end{aligned}$$

$$\begin{aligned}\mathbf{D} &= \epsilon_0\mathbf{E}+\mathbf{P}\\ \mathbf{H} &= \mathbf{B}/\mu_0-\mathbf{M}\end{aligned}$$

electric polarization, i.e., density of electric dipoles

magnetization, i.e., density of magnetic dipoles

It is not quite clear how to define $\mathbf{D}$ or $\mathbf{P}$ for plasma, i.e. for a set of free charges!

However the polarization current: $\mathbf{J}_P = \frac{\partial\mathbf{P}}{\partial t}$

and the magnetization current: $\mathbf{J}_M = \nabla\times\mathbf{M}$

are useful plasma concepts.

Electric and magnetic fields are empirically determined through the Lorentz force

$\boxed{\mathbf{F} = \frac{d\mathbf{p}}{dt} = q(\mathbf{E}+\mathbf{v}\times\mathbf{B})}$ or $\mathbf{F} = \frac{d}{dt}(\gamma m\mathbf{v}) = q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$

Because $\mathbf{v}\cdot\mathbf{F} = q(\mathbf{v}\cdot\mathbf{E}+\mathbf{v}\cdot\mathbf{v}\times\mathbf{B}) = q(\mathbf{v}\cdot\mathbf{E})$

only $\mathbf{E}$ performs work $\frac{dW}{dt} = \frac{d}{dt}(\gamma mc^2) = q\mathbf{E}\cdot\mathbf{v}$

It is often stated that a changing magnetic field can accelerate particles, but actually particles are then accelerated by the induced electric field!

Constitutive equations

Ohm's law $\boxed{\mathbf{J} = \sigma \cdot \mathbf{E}}$ relating the electric current and electric field is similar to the other constitutive equations $\mathbf{D} = \epsilon \cdot \mathbf{E}$ and $\mathbf{B} = \mu \cdot \mathbf{H}$

The conductivity σ, permittivity ε, and permeability μ depend on the electric and magnetic properties of the media considered. They may be scalars or tensors, and there does not need to be a local constitutive relation at all, not even Ohm's law!

A medium is called linear if ε, μ, σ are scalars and they are not functions of time and space.

Note that also in linear media $\varepsilon = \varepsilon(\omega,\mathbf{k})$.
This is a key relationship in plasma physics!

Scalar and vector potentials

How to solve Maxwell's equations?

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \exists \mathbf{A} \; : \; \mathbf{B} = \nabla \times \mathbf{A}$$

$\mathbf{A}$ is called vector potential

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} \right) = 0$$

thus $\mathbf{E} = -\partial \mathbf{A}/\partial t - \nabla \varphi$

scalar potential

Inserting these into remaining Maxwell's equations we get

$$\begin{aligned} \nabla^2 \varphi + \frac{\partial(\nabla \cdot \mathbf{A})}{\partial t} &= -\rho/\epsilon_0 \\ \nabla^2 \mathbf{A} - \frac{1}{c^2}\frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla \left(\nabla \cdot \mathbf{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} \right) &= -\mu_0 \mathbf{J} \end{aligned}$$

By solving $\mathbf{A}$ and φ from these we get $\mathbf{E}$ and $\mathbf{B}$ as their derivatives

Because $\mathbf{E}$ and $\mathbf{B}$ are derivatives of $\mathbf{A}$ and φ, we have certain freedom to manipulate them and yet get the same physical fields

Gauge transformations:

$$\begin{aligned} \mathbf{A} &\rightarrow \mathbf{A}' = \mathbf{A} + \nabla \Psi \\ \varphi &\rightarrow \varphi' = \varphi - \partial \Psi/\partial t \end{aligned}$$

The Lorenz gauge is defined by condition

$$\boxed{\nabla \cdot \mathbf{A}' + \frac{1}{c^2}\frac{\partial \varphi'}{\partial t} = 0} \Rightarrow \boxed{\begin{aligned} \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\varphi &= -\rho/\epsilon_0 \\ \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} &= -\mu_0 \mathbf{J} \end{aligned}}$$

Four inhomogeneous wave equations, for which there are well-known solution methods leading to retarded potentials (c.f. any good ED text-book!)

The potentials take into account the finite speed of information (c) in a relativistically correct way. In 4-vector notation the wave equation is

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)A^\alpha = -\mu_0 j^\alpha \quad \text{where} \quad A^\alpha = (\varphi/c, \mathbf{A}) \ \ \& \ \ j^\alpha = (c\rho, \mathbf{J})$$

From the potentials (exercise) $\Rightarrow$

$$\begin{aligned} \mathbf{E} &= \frac{1}{4\pi\epsilon_0}\left\{\int \frac{[\rho]\mathbf{R}}{R^3}\,d^3r' + \frac{1}{c}\int\left(\frac{2[\mathbf{J}]\cdot\mathbf{R}\mathbf{R}}{R^4} - \frac{[\mathbf{J}]}{R^2}\right)d^3r' \right. \\ &\quad \left. + \frac{1}{c^2}\int\left(\frac{([\dot{\mathbf{J}}]\times\mathbf{R})\times\mathbf{R}}{R^3}\right)d^3r'\right\} \\ \mathbf{B} &= \frac{\mu_0}{4\pi}\left\{\int \frac{[\mathbf{J}]\times\mathbf{R}}{R^3}\,d^3r' + \frac{1}{c}\int\frac{[\dot{\mathbf{J}}]\times\mathbf{R}}{R^2}\,d^3r'\right\}, \end{aligned}$$

Taking only terms vanishing as $1/R$ we get the radiation fields

$$\begin{aligned} \mathbf{E}_{rad} &= \frac{1}{4\pi\epsilon_0 c^2}\int \frac{([\dot{\mathbf{J}}]\times\mathbf{R})\times\mathbf{R}}{R^3}\,d^3r' \\ \mathbf{B}_{rad} &= \frac{1}{4\pi\epsilon_0 c^3}\int \frac{[\dot{\mathbf{J}}]\times\mathbf{R}}{R^2}\,d^3r'. \end{aligned}$$

The Coulomb gauge condition is $\boxed{\nabla \cdot \mathbf{A}' = 0}$

$$\varphi = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r}',t)}{R}\,d^3r' \qquad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = \frac{1}{c^2}\nabla\frac{\partial\varphi}{\partial t} - \mu_0\mathbf{J}$$

$$\mathbf{J} = \mathbf{J}_l + \mathbf{J}_t\ ;\ \nabla\times\mathbf{J}_l = 0\ ;\ \nabla\cdot\mathbf{J}_t = 0 \qquad \nabla^2\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\mu_0\mathbf{J}_t$$

$$\mathbf{A}(\mathbf{r},t) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}_t(\mathbf{r}', t-R/c)}{R}\,d^3r'$$

The Coulomb gauge is very useful in radiation problems, because the radiation fields can then be calculated from the vector potential alone.

The Coulomb gauge separates the static and inductive electric fields

$\mathbf{E}_s = -\nabla\varphi\ ;\ \mathbf{E}_i = -\partial\mathbf{A}/\partial t$ but this separation is not Lorentz covariant (be careful with moving frames of reference!)

If there are no local currents, i.e., the magnetic field is determined by external sources only, the field can be expressed as a gradient of a magnetic scalar potential

$$\nabla \times \mathbf{B} = 0 \Rightarrow \mathbf{B} = -\nabla\psi$$

$$\nabla \cdot \mathbf{B} = 0 \Rightarrow \nabla^2\psi = 0$$

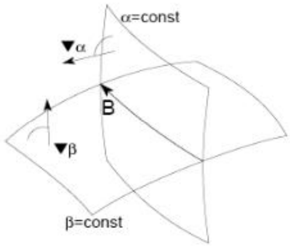
This greatly simplifies the calculations because the methods of potential theory, similar to electrostatics become available. Even the rather complicated magnetic field of the Earth can be expressed using spherical harmonic expansions

Another presentation of $\mathbf{B}$ is to use the Euler potentials α, β, χ defined by

$$\mathbf{A} = \alpha\nabla\beta + \nabla\chi \quad \Rightarrow$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \nabla \times (\alpha\nabla\beta + \nabla\chi) = \nabla\alpha \times \nabla\beta$$

Thus $\mathbf{B}$ is perpendicular to both $\nabla\alpha$ and $\nabla\beta$, i.e., α and β are constant along the magnetic field lines. This is useful when one needs to trace a field line from one location to another.



Conservation of EM energy
Poynting's theorem

The energy of electromagnetic field is given by

$$W_{EM} = \frac{1}{2}\int (\mathbf{E}\cdot\mathbf{D} + \mathbf{H}\cdot\mathbf{B})\, d^3r \qquad w_E = \frac{1}{2}\mathbf{E}\cdot\mathbf{D} \qquad w_M = \frac{1}{2}\mathbf{H}\cdot\mathbf{B}$$

Strating from Maxwell's equations it is a straightforward exercise to get

$$\boxed{\nabla\cdot\mathbf{S} = -\mathbf{E}\cdot\mathbf{J} - \mathbf{E}\cdot\partial\mathbf{D}/\partial t - \mathbf{H}\cdot\partial\mathbf{B}/\partial t}$$ Poynting's theorem

where $\boxed{\mathbf{S} = \mathbf{E}\times\mathbf{H}}$ is the Poynting vector

Integrating over volume V (and using Gauss's law for the divergence)

$$\underbrace{-\int_V \mathbf{J}\cdot\mathbf{E}\, d^3r}_{\text{work performed by the EM field}} = \underbrace{\oint_{\partial V} \mathbf{S}\cdot d\mathbf{a}}_{\text{energy flux through the surface of } V} + \underbrace{\int_V \frac{\partial}{\partial t}(w_E + w_M)\, d^3r}_{\text{change of energy in } V}$$

Conservation law of electromagnetic energy

Guiding centre approximation

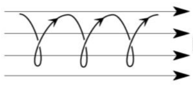
Equation of motion of charged particles is

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}_{non-EM}$$

(assume, for the time being, nonrelativistic motion; $\gamma = 1$ and $\mathbf{p} = m\mathbf{v}$)

Consider the case $\boxed{\mathbf{E} = 0 \text{ and } \mathbf{B} = \text{const}}$ (neglect the non-EM forces)

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B})$$



$$\boxed{\omega_c = \frac{qB}{m}}$$

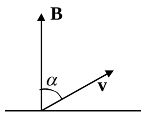
cyclotron frequency
gyro frequency
Larmor frequency

The radius of the circle is (Larmor radius) $r_L = \dfrac{v_\perp}{|\omega_c|} = \dfrac{mv_\perp}{|q|B}$; $v_\perp = \sqrt{v_x^2 + v_y^2}$

The gyro period (cyclotron period, Larmor time) is: $\tau_L = \dfrac{2\pi}{|\omega_c|}$

The pitch angle (α) of the helical path is defined by

$$\tan\alpha = v_\perp / v_\parallel$$
$$\alpha = \arcsin(v_\perp / v) = \arccos(v_\parallel / v)$$



The frame of reference where $v_\parallel = 0$: Guiding centre system (GCS)

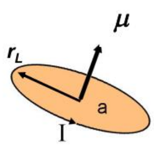
Decomposition of the motion to the motion of the guiding centre and to the gyro motion is called the **guiding centre approximation**

In the GCS the charge causes an electric current: $I = q / \tau_L$

The magnetic moment associated with the circular loop is

$$\boxed{\mu = I\pi r_L^2 = \frac{1}{2}\frac{q^2 r_L^2 B}{m} = \frac{1}{2}\frac{mv_\perp^2}{B} = \frac{W_\perp}{B}}$$

or, in the vector form $\boxed{\boldsymbol{\mu} = \frac{1}{2} q\, \mathbf{r}_L \times \mathbf{v}_\perp}$



Clearly: $\boldsymbol{\mu}$ is always opposite to $\mathbf{B}$ ($\mathbf{r}_L$ depends on the sign of q)

Thus plasma can be considered a diamagnetic medium:

Placing a large number of charged particles to external magnetic field reduces the field.

E x B drift

Let $\mathbf{E}$ = const and $\mathbf{B}$ = const

The eq. of motion along $\mathbf{B}$ is $m\dot{v}_{\|} = qE_{\|}$

→ constant acceleration parallel/antiparallel to $\mathbf{B}$

→ very rapid cancellation of large-scale $\mathbf{E}_{\|}$ in plasma (important)

The perpendicular components of the eq. of motion are

$$\begin{aligned} \dot{v}_x &= \omega_c v_y + \frac{q}{m}E_x \\ \dot{v}_y &= -\omega_c v_x \,. \end{aligned} \quad \Rightarrow \quad \begin{aligned} \ddot{v}_x &= -\omega_c^2 v_x \\ \ddot{v}_y &= -\omega_c^2 \left(v_y + \frac{E_x}{B} \right) \end{aligned}$$

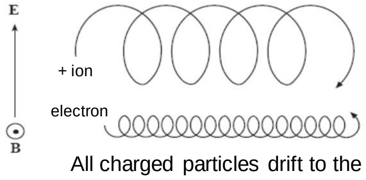
Substitution $v_y' = v_y + E_x/B$ leads again to gyro motion but now the GC drifts in the y-direction with speed E_x/B

In vector form: $\boxed{\mathbf{v}_E = \frac{\mathbf{E}\times\mathbf{B}}{B^2}}$

Same as making Lorentz transformation to GCS:

$\mathbf{E}' = \mathbf{E} + \mathbf{w}\times\mathbf{B}$ (non-relat. $\gamma = 1$)

where $\mathbf{E}' = 0,\ \mathbf{E} = -\mathbf{w}\times\mathbf{B}$



All charged particles drift to the same direction $\perp\mathbf{E}$ and $\perp\mathbf{B}$

Other non-magnetic drifts

Write the perpendicular eq. of motion in the form $\frac{d\mathbf{v}_\perp}{dt} = \frac{q}{m}(\mathbf{v}_\perp \times \mathbf{B}) + \frac{\mathbf{F}_\perp}{m}$

Assume that $\mathbf{F}_\perp$ gives rise to a drift $\mathbf{v}_D$ and transform $\mathbf{v}_\perp = \mathbf{v}'_\perp + \mathbf{v}_D$

$$\Rightarrow \frac{d\mathbf{v}'_\perp}{dt} = \frac{q}{m}(\mathbf{v}'_\perp \times \mathbf{B}) + \frac{q}{m}(\mathbf{v}_D \times \mathbf{B}) + \frac{\mathbf{F}_\perp}{m}$$

In GCS the last two terms must sum to 0 $\Rightarrow$ $\boxed{\mathbf{v}_D = \frac{\mathbf{F}_\perp \times \mathbf{B}}{qB^2}}$ $(*)$

This requires $F/qB \ll c$. If $F > qcB$, the GC approximation cannot be used!

Inserting $\mathbf{F}_\perp = q\mathbf{E}$ into $(*)$ we get the ExB-drift

$\mathbf{F}_\perp = m\mathbf{g}$ gives the gravitational drift $\boxed{\mathbf{v}_g = \frac{m\mathbf{g}\times\mathbf{B}}{qB^2} \propto \frac{m}{q}}$ separates charges → current

Slow time variations in $\mathbf{E}$ → polarization drift $\boxed{\mathbf{v}_P = \frac{1}{\omega_c B}\frac{d\mathbf{E}_\perp}{dt}}$

The corresponding polarization current is

$$\mathbf{J}_P = n_e e(\mathbf{v}_{Pi} - \mathbf{v}_{Pe}) = \frac{n_e(m_i + m_e)}{B^2}\frac{d\mathbf{E}_\perp}{dt} \simeq \frac{n_e m_i}{B^2}\frac{d\mathbf{E}_\perp}{dt}$$

carried by ions!