Session of Monday 10 November at 16.00-17.30 in aud A315.

1. On Vdh II.19 the matrix element of the energy-momentum tensor $T^{\mu\nu} = T^{\nu\mu}$ between nucleons is expanded in terms of three Lorentz-invariant form factors $A(t)$, $B(t)$ and $C(t)$ as

$$\langle p + \tfrac{1}{2}\Delta | T^{\mu\nu}(0) | p - \tfrac{1}{2}\Delta \rangle = \\ = \bar{u}(p + \tfrac{1}{2}\Delta) \left[A(t)\gamma^{(\mu} p^{\nu)} + B(t) p^{(\mu} i\sigma^{\nu)\alpha} \frac{\Delta_\alpha}{2m} + C(t)(\Delta^\mu \Delta^\nu - \Delta^2 g^{\mu\nu}) \frac{1}{m} \right] u(p - \tfrac{1}{2}\Delta)$$

where $t = \Delta^2$.

   (a) Show that all three terms in this expansion are consistent with $\partial_\mu T^{\mu\nu}(x) = 0$.

   (b) Show that a term like $D(t)\gamma^{(\mu}\Delta^{\nu)}$ is not allowed.

2. On Vdh II.4 the Light-Front matrix element is expressed in terms of the GPD's H and E as

$$\frac{p^+}{2\pi} \int dy^- e^{ixp^+ y^-/2} \langle p + \tfrac{1}{2}\Delta, \lambda' | \bar{q}(-\tfrac{1}{2}y) \gamma \cdot n \, q(\tfrac{1}{2}y) | p - \tfrac{1}{2}\Delta, \lambda \rangle_{y^+ = y_\perp = 0} = \\ = \bar{u}(p + \tfrac{1}{2}\Delta, \lambda') \left[H(x, \xi, t) \gamma \cdot n + E(x, \xi, t) i\sigma^{\mu\nu} \frac{\Delta_\nu}{2m} n_\mu \right] u(p - \tfrac{1}{2}\Delta, \lambda) \tag{1}$$

where $\gamma \cdot n = \gamma^+$, $\Delta^+ = -2\xi p^+$ and $t = \Delta^2$.

   (a) In DVCS ($\gamma^* p \to \gamma p$), the momentum transfer from the target which is kinematically required for the $\gamma^* \to \gamma$ transition is $-\Delta^+$. Show that $\xi = x_B/(2 - x_B)$, where $x_B = Q^2/2m\nu$ is the standard Bjorken variable.

   (b) Explain why there are no other Lorentz structures on the rhs. of (1), such as $p \cdot n$ or $\Delta \cdot n$. *Hint:* Compare with the evaluation of the form factors in problem 1b of Ex. 7.

3. The electron contribution to the energy-momentum tensor in QED is

$$T^{\mu\nu} = i\tfrac{1}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - g^{\mu\nu}\bar{\psi}(i\not{D} - m)\psi$$

where ψ is the electron field and $iD^\mu = i\partial^\mu - eA^\mu$. Verify by an explicit calculation that

$$\langle e(q, \lambda') | \int d^3\boldsymbol{x} T^{0\nu}(\boldsymbol{x}) | e(p, \lambda) \rangle = p^\nu \langle e(q, \lambda') | e(p, \lambda) \rangle$$

where $|e(p, \lambda)\rangle$ is the state of an electron with momentum p and helicity λ.