Session of Monday 10 November at 16.00-17.30 in aud A315.

1. On Vdh II.19 the matrix element of the energy-momentum tensor $T^{\mu\nu} = T^{\nu\mu}$ between nucleons is expanded in terms of three Lorentz-invariant form factors $A(t)$, $B(t)$ and $C(t)$ as

$$\langle p + \tfrac{1}{2}\Delta | T^{\mu\nu}(0) | p - \tfrac{1}{2}\Delta \rangle =$$
$$= \bar{u}(p + \tfrac{1}{2}\Delta) \left[A(t)\gamma^{(\mu}p^{\nu)} + B(t)p^{(\mu}i\sigma^{\nu)\alpha}\frac{\Delta_\alpha}{2m} + C(t)(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})\frac{1}{m} \right] u(p - \tfrac{1}{2}\Delta)$$

where $t = \Delta^2$.

   (a) Show that all three terms in this expansion are consistent with $\partial_\mu T^{\mu\nu}(x) = 0$.

   (b) Show that a term like $D(t)\gamma^{(\mu}\Delta^{\nu)}$ is not allowed.

2. On Vdh II.4 the Light-Front matrix element is expressed in terms of the GPD's H and E as

$$\frac{p^+}{2\pi}\int dy^- e^{ixp^+y^-/2} \langle p + \tfrac{1}{2}\Delta, \lambda' | \bar{q}(-\tfrac{1}{2}y)\gamma\cdot n\, q(\tfrac{1}{2}y) | p - \tfrac{1}{2}\Delta, \lambda \rangle_{y^+ = y_\perp = 0} = \tag{1}$$
$$= \bar{u}(p + \tfrac{1}{2}\Delta, \lambda') \left[H(x,\xi,t)\gamma\cdot n + E(x,\xi,t) i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m}n_\mu \right] u(p - \tfrac{1}{2}\Delta, \lambda)$$

where $\gamma\cdot n = \gamma^+$, $\Delta^+ = -2\xi p^+$ and $t = \Delta^2$.

   (a) In DVCS ($\gamma^* p \to \gamma p$), the momentum transfer from the target which is kinematically required for the $\gamma^* \to \gamma$ transition is $-\Delta^+$. Show that $\xi = x_B/(2 - x_B)$, where $x_B = Q^2/2m\nu$ is the standard Bjorken variable.

   (b) Explain why there are no other Lorentz structures on the rhs. of (1), such as $p\cdot n$ or $\Delta\cdot n$. *Hint:* Compare with the evaluation of the form factors in problem 1b of Ex. 7.

3. The electron contribution to the energy-momentum tensor in QED is

$$T^{\mu\nu} = i\tfrac{1}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - g^{\mu\nu}\bar{\psi}(i\not{D} - m)\psi$$

where ψ is the electron field and $iD^\mu = i\partial^\mu - eA^\mu$. Verify by an explicit calculation that

$$\langle e(q,\lambda') | \int d^3\boldsymbol{x}\, T^{0\nu}(\boldsymbol{x}) | e(p,\lambda) \rangle = p^\nu \langle e(q,\lambda') | e(p,\lambda) \rangle$$

where $|e(p,\lambda)\rangle$ is the state of an electron with momentum p and helicity λ.

QCD 10/08

1. $\langle p+\tfrac{1}{2}\Delta|T^{\mu\nu}(x)|p-\tfrac{1}{2}\Delta\rangle = e^{ix\cdot\Delta}\langle p+\tfrac{1}{2}\Delta|T^{\mu\nu}(0)|p-\tfrac{1}{2}\Delta\rangle$

$\Rightarrow \langle p+\tfrac{1}{2}\Delta|\partial_\mu T^{\mu\nu}(x)|p-\tfrac{1}{2}\Delta\rangle = e^{ix\cdot\Delta}\, i\Delta_\mu \langle p+\tfrac{1}{2}\Delta|T^{\mu\nu}(0)|p-\tfrac{1}{2}\Delta\rangle = 0$

a) Multiplying the rhs by Δ_μ and using

$\bar{u}(p+\tfrac{1}{2}\Delta)\,\not\Delta\, u(p-\tfrac{1}{2}\Delta) = \bar{u}(p+\tfrac{1}{2}\Delta)\left[(\not p+\tfrac{1}{2}\not\Delta)-(\not p-\tfrac{1}{2}\not\Delta)\right]u(p-\tfrac{1}{2}\Delta)$

$= \bar{u}(p+\tfrac{1}{2}\Delta)\left[m-m\right]u(p-\tfrac{1}{2}\Delta) = 0$

$(p\pm\tfrac{1}{2}\Delta)^2 = p^2 \pm p\cdot\Delta + \tfrac{1}{4}\Delta^2 = m^2 \quad\Rightarrow\quad p\cdot\Delta = 0$

we find

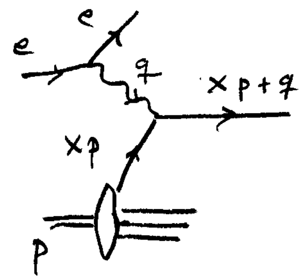
$$\bar{u}(p+\tfrac{1}{2}\Delta)\Big\{\tfrac{1}{2}A(t)\Big[\overset{\overset{0}{\uparrow}}{\not\Delta}\, p^\nu + \gamma^\nu \overset{\overset{0}{\uparrow}}{p\cdot\Delta}\Big] + \tfrac{i}{2}B(t)\Big[\overset{\overset{0}{\uparrow}}{p\cdot\Delta}\,\sigma^{\nu\alpha}\frac{\Delta_\alpha}{2m}$$

$$+\, p^\nu \overset{\overset{0}{\uparrow}}{\Delta_\mu\sigma^{\mu\alpha}}\frac{\Delta_\alpha}{2m}\Big] + \frac{1}{m}C(t)\left[\Delta^2\Delta^\nu - \Delta^2\Delta^\nu\right]\Big\}u(p-\tfrac{1}{2}\Delta) = 0$$

b) $$\bar{u}(p+\tfrac{1}{2}\Delta)\Big\{\tfrac{1}{2}D(t)\Big[\overset{\overset{0}{\uparrow}}{\not\Delta}\,\Delta^\nu + \overset{\overset{0}{\not\uparrow}}{\gamma^\nu\Delta^2}\Big]\Big\}u(p-\tfrac{1}{2}\Delta) \neq 0$$

Similarly, any other L-covariant expression can (presumably) be excluded, or expressed in terms of A, B, C.

2\. a) In ordinary DIS, $x_B p^+$ is the momentum transfer required for the $\gamma^* \to q$ transition:

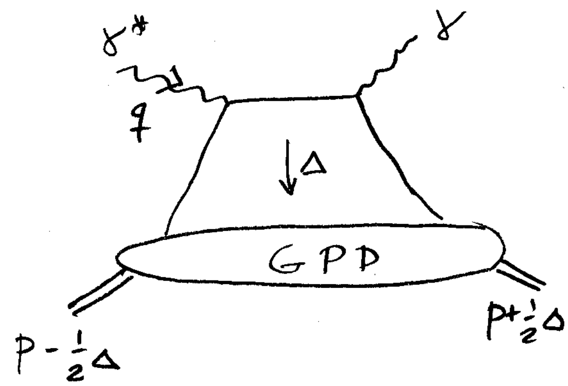


$$(xp+q)^2 = m_q^2 \simeq 0$$

$$2x\, p\cdot q + q^2 = 0, \quad x = \frac{Q^2}{2p\cdot q} \equiv x_B$$

Note: $x = x_B$ is Lorentz-invariant!
In the target rest frame: $2p\cdot q = 2m\nu$

DVCS: $-\Delta^+ = 2\xi p^+$ is the '+' momentum transfer from the target. Now target is not at rest



$$(q-\Delta)^2 = m_\gamma^2 = 0$$

$$q^2 - q^-\Delta^+ \simeq 0 \quad \Rightarrow \quad -\Delta^+ = \frac{Q^2}{q^-} = 2\xi p^+; \quad 2\xi = \frac{Q^2}{p^+q^-}$$

$$x_B = \frac{Q^2}{2(p-\frac{1}{2}\Delta)\cdot q} = \frac{Q^2}{p^+(1+\xi)q^-} = \frac{2\xi}{1+\xi}$$

$$\xi(2-x_B) = x_B\;; \quad \xi = \frac{x_B}{2-x_B}$$

Note: ξ is a Lorentz invariant!
Hence GPD's $H(x,\xi,t)$ etc are L-invariant

2b) The matrix element depends on the helicities $\lambda, \lambda' = \pm\frac{1}{2}$ of the initial & final protons.
$\Rightarrow$ 4 invariant functions can be independent.

Parity relates matrix element λ, λ' to $-\lambda, -\lambda'$
$\Rightarrow$ 2 parity conserving invariant functions

Lorentz structures $p \cdot n$ and $\Delta \cdot n$ would not give independent GPD's

Cf. calculation of form factors : QCD 7/08 # 1b

Eg., contribution to $\lambda = \lambda'$ is proportional to

$$T_{\pm\pm} \propto \mathrm{Tr}\left[(\not{p} + \tfrac{1}{2}\not{\Delta} + m)[\not{n} \text{ or } n\cdot p \text{ or } n\cdot\Delta](\not{p} - \tfrac{1}{2}\not{\Delta} + m)\not{n}(1 \pm \gamma_5)\right]$$

where γ_5 does not contribute in $\vec{P}_\perp = 0$ frame

$\Rightarrow T_{++} = T_{--}$, similarly $T_{+-} = -T_{-+}$ $(\varphi = 0)$

The opposite behavior under parity is given by $\tilde{H}$ and $\tilde{E}$, which have a γ_5 in Lorentz structure.

Altogether, $H, E, \tilde{H}, \tilde{E}$ is a complete basis of 4 functions for 4 helicity combinations.

3. $|e(p,\lambda)\rangle = b^+(p,\lambda)|0\rangle \qquad (p^0 = E_p)$

$$\{b(q,\lambda'),\ b^+(p,\lambda)\} = 2E_p(2\pi)^3\delta^3(\vec{p}-\vec{q})\,\delta_{\lambda\lambda'}$$

$$\Rightarrow \langle q,\lambda'|p,\lambda\rangle = \qquad \text{—— " ——}$$

$$\psi(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2E_k}\sum_{\lambda''}\left[b(k,\lambda'')u(k,\lambda'')e^{-ik\cdot x} + d^+(k,\lambda'')v(k,\lambda'')e^{ik\cdot x}\right]$$

$A(x) =$ " $\quad a(\ )\ldots + a^+(\ )$ does not contribute

$\Rightarrow iD^\mu \to i\partial^\mu$

$$i\partial^\nu\psi(x) = \int \frac{d^3\vec{k}}{(2\pi)^3 2E_k}\sum_{\lambda''}\left[b(k,\lambda'')u(k,\lambda'')k^\nu e^{-ik\cdot x} - d^+ u\, k^\nu e^{ik\cdot x}\right]$$

$$\langle e(q,\lambda')|\int d^3\vec{x}\, T^{0\nu}(x)|e(p,\lambda)\rangle = \bar{u}(q,\lambda')\left[\tfrac{1}{2}\gamma^0 p^\nu + \tfrac{1}{2}p^0\gamma^\nu - \right.$$

$$\left. - g^{0\nu}\overbrace{(\not{p}-m)}^{\to 0}\right]u(p,\lambda)\int d^3\vec{x}\, e^{-ip\cdot x + iq\cdot x} =$$

$$= \frac{1}{2}\bar{u}(p,\lambda')\left[\gamma^0 p^\nu + p^0\gamma^\nu\right]u(p,\lambda)(2\pi)^3\delta^3(\vec{p}-\vec{q})$$

Now $\bar{u}(p,\lambda')\gamma^\nu u(p,\lambda) = 2p^\nu\delta_{\lambda\lambda'}$ (Gordon identity)
$\quad\ \bar{v}(p,\lambda')\gamma^\nu v(p,\lambda) = -$ " —— " ——

$$\Rightarrow \langle e(q,\lambda')|\int d^3\vec{x}\, T^{0\nu}(x)|e(p,\lambda)\rangle = p^\nu \underbrace{2p^0(2\pi)^3\delta^3(\vec{p}-\vec{q})\,\delta_{\lambda\lambda'}}_{\langle e(q,\lambda')|e(p,\lambda)\rangle}$$

There is also a contribution from $\{d, d^+\}$:

$$\frac{1}{2}\int \frac{d^3\vec{k}}{(2\pi)^3 2E_k}\sum_{\lambda''}\bar{v}(k,\lambda'')\left[\gamma^0(-k^\nu) + (-k^0)\gamma^\nu\right]v(k,\lambda'')\int d^3\vec{x}\cdot\langle q,\lambda'|p,\lambda\rangle$$

$$= -2m\int\frac{d^3\vec{k}}{(2\pi)^3}k^\nu\int d^3\vec{x}\cdot\langle q,\lambda'|p,\lambda\rangle = \text{"}\infty\text{"}$$

Apparently we should subtract this contribution by normal ordering the operators: $\quad :T^{\mu\nu}:$