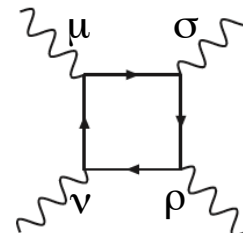


Session of Monday 17 November at 16.00-17.30 in aud A315.

1. Kubis p. I.12 gives the effective low-energy lagrangian corresponding to the light-by-light scattering diagram below. Show that this loop diagram $L^{\mu\nu\rho\sigma}$ (which is symmetric in the Lorentz indices μ, ν, ρ, σ of the interaction vertices) is UV finite. This is required by renormalizability since QED has no 4-photon coupling.

Hint: The only potentially divergent contributions arise from the leading term in the loop momentum for all propagators. Consider the possible structure of Lorentz indices, then simplify the Dirac algebra by summing over some Lorentz indices.



2. (a) Derive the expressions for the vector and axial vector Noether currents of the lagrangian $\mathcal{L} = \frac{F^2}{4} \langle (\partial_\mu U) \partial^\mu U^\dagger \rangle$ given on Kubis I.24:

$$V_a^\mu = i \frac{F^2}{4} \langle \lambda_a [\partial^\mu U, U^\dagger] \rangle \qquad A_a^\mu = i \frac{F^2}{4} \langle \lambda_a \{ \partial^\mu U, U^\dagger \} \rangle$$

- (b) Using this result and $U = \exp(i\phi/F)$ show that to first order in ϕ ,

$$\langle 0 | A_a^\mu(x) | \phi_b(p) \rangle = i p^\mu e^{-ip \cdot x} \delta_{ab} F$$

3. The effective four-fermion interaction governing pion decay is

$$\mathcal{L}(x) = \frac{G_F}{\sqrt{2}} V_{ud} \bar{d}(x) \gamma_\alpha (1 - \gamma_5) u(x) \bar{\nu}_\mu(x) \gamma^\alpha (1 - \gamma_5) \mu(x) \tag{1}$$

where the Fermi coupling $G_F = 1.16639 \cdot 10^{-5} \text{ GeV}^{-2}$, the CKM matrix element $V_{ud} = 0.9753$ and the quark and lepton fields are as indicated.

- (a) Calculate the pion decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ matrix element $T = \langle \mu^+(k) \nu(q) | \int d^4x \mathcal{L}(x) | \pi^+(p) \rangle$ making use of

$$\langle 0 | \bar{d}(x) \gamma^\mu \gamma_5 u(x) | \pi^+(p) \rangle = -i \sqrt{2} f_\pi p^\mu e^{-ip \cdot x} \tag{2}$$

where the pion decay constant $f_\pi = 93 \text{ MeV}$.

- (b) Using the matrix element and the expression for the decay width given in the Feynman rules of Peskin, derive

$$\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu) = \frac{G_F^2 V_{ud}^2 f_\pi^2}{4\pi m_\pi^3} m_\mu^2 (m_\pi^2 - m_\mu^2)^2 \tag{3}$$

and compare with the experimental value of the π^+ life-time, $\tau_\pi = 2.60 \cdot 10^{-8} \text{ s}$.

- (c) Calculate the ratio $\Gamma(\pi^+ \rightarrow e^+ \nu_e) / \Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)$, and compare with the experimental value $1.23 \cdot 10^{-4}$. Explain the smallness of this ratio, *i.e.*, why the decay width vanishes with the charged lepton mass.