Session of Monday 24 November at 16.00-17.30 in aud A315.

1. (a) Verify the statement on Kubis p. II.10 that the ϕ_3 and ϕ_8 fields in the chiral lagrangian mix according to

$$\begin{aligned}\mathcal{L}^{(2)} &= \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U\partial^\mu U^\dagger + 2B\mathcal{M}U^\dagger + \mathcal{M}^\dagger U\right] \\ &\supset -\frac{1}{2}B\begin{pmatrix}\phi_3 & \phi_8\end{pmatrix}\begin{pmatrix} m_u+m_d & (m_u-m_d)/\sqrt{3} \\ (m_u-m_d)/\sqrt{3} & (m_u+m_d+4m_s)/3\end{pmatrix}\begin{pmatrix}\phi_3 \\ \phi_8\end{pmatrix}\end{aligned}$$

   (b) Show that the eigenvalues of the matrix in (a) determine the masses $m_{\pi^0}^2 = B(m_u+m_d)$ and $m_\eta^2 = B(m_u+m_d+4m_s)/3$, up to corrections of $\mathcal{O}\left((m_u-m_d)^2\right)$.

2. (a) Show using the classical equations of motion of the QCD lagrangian with a single masselss quark mass that $\partial_\mu j_5^\mu = 0$, where the axial vector current $j_5^\mu(x) = \bar\psi(x)\gamma^\mu\gamma_5\psi(x)$. Find the expression for $\partial_\mu j_5^\mu$ also when the quark mass $m_q \neq 0$.

   (b) The Adler-Bell-Jackiw anomaly

$$\partial_\mu j_5^\mu = -\frac{e^2}{16\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$$

   states that the axial vector current is not conserved at the operator (quantum) level even when the quark mass vanishes. Show that the anomaly implies

$$\Delta N_R - \Delta N_L = \frac{e^2}{2\pi^2}\int d^4x\, \boldsymbol{E}\cdot\boldsymbol{B}$$

   where $N_R = \int d^3\boldsymbol{x}\psi_R^\dagger\psi_R$ is the total number of right-handed fermions, and similarly for L. Δ refers to the change in number from $x^0 = -\infty$ to $x^0 = +\infty$, and $\boldsymbol{E}, \boldsymbol{B}$ are the electric and magnetic fields. [This is problem 19.1 of Peskin and Schroeder.]

3. The $\pi^0 \to \gamma + \gamma$ decay width can be calculated from the Bell-Jackiw anomaly as described in, *e.g.*, section 19.3 of Peskin and Schroeder. The $j_5^{\mu 3}$ component of the axial current, which can annihilate a π^0, has the electromagnetic anomaly

$$\partial_\mu j_5^{\mu 3} = -\frac{e^2}{32\pi^2}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} \tag{1}$$

   where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the QED field strength tensor. The matrix element of the current between the vacuum and a state with 2 photons of momenta p and k (helicities not indicated explicitly) has the form

$$\langle p,k|j_5^{\mu 3}(q)|0\rangle = \varepsilon_\nu^*(p)\varepsilon_\lambda^*(k)\mathcal{M}^{\mu\nu\lambda}(p,k) \tag{2}$$

PTO

(a) Verify that the decomposition of the amplitude

$$\begin{aligned}\mathcal{M}^{\mu\nu\lambda}(p,k) &= q^\mu \epsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta \mathcal{M}_1 + (\epsilon^{\mu\nu\alpha\beta} k^\lambda - \epsilon^{\mu\lambda\alpha\beta} p^\nu) k_\alpha p_\beta \mathcal{M}_2 \\ &+ [(\epsilon^{\mu\nu\alpha\beta} p^\lambda - \epsilon^{\mu\lambda\alpha\beta} k^\nu) k_\alpha p_\beta - \epsilon^{\mu\nu\lambda\sigma}(p-k)_\sigma p\cdot k] \mathcal{M}_3 \end{aligned} \tag{3}$$

where the invariant amplitudes $\mathcal{M}_i = \mathcal{M}_i(q^2)$, is consistent with the constraints

(i) $p_\nu \mathcal{M}^{\mu\nu\lambda}(p,k) = k_\lambda \mathcal{M}^{\mu\nu\lambda}(p,k) = 0$ (gauge invariance);
(ii) $\mathcal{M}^{\mu\nu\lambda}(p,k) = \mathcal{M}^{\mu\lambda\nu}(k,p)$ (identical photons);
(iii) $\mathcal{M}^{\mu\nu\lambda}(p,k) = \mp \mathcal{M}^{\mu\nu\lambda}(k,p)$, with $-(+)$ for $\mu = 0$ ($\mu = 1,2,3$) (parity invariance in the $\boldsymbol{q} = 0$ frame since $\boldsymbol{p} = -\boldsymbol{k}$ and the polarization (axial) vectors do not change under parity).

(b) Show that (3) implies

$$iq_\mu \mathcal{M}^{\mu\nu\lambda}(p,k) = iq^2 \epsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta (\mathcal{M}_1 + \mathcal{M}_3) \tag{4}$$

and that using (1) one gets

$$iq_\mu \mathcal{M}^{\mu\nu\lambda}(p,k) = \langle p,k|\partial_\mu j_5^{\mu 3}(q)|0\rangle = -\frac{e^2}{4\pi^2} \epsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta \tag{5}$$

(c) Either $\mathcal{M}_1$ or $\mathcal{M}_3$ must have a $1/q^2$ pole for (4) and (5) to be compatible. Only the massless π^0 propagator in $\langle p,k|S|\pi^0\rangle\langle \pi^0|j_5^{\mu 3}(q)|0\rangle$ can give this pole. Using $\langle \pi^0|j_5^{\mu 3}(x)|0\rangle = ip^\mu f_\pi \exp(ip\cdot x)$ and writing the pion decay amplitude

$$i\mathcal{M}(\pi^0 \to 2\gamma) \equiv \langle p,k|S|\pi^0\rangle = iA\, \varepsilon_\nu^*(p) \varepsilon_\lambda^*(k)\, \epsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta \tag{6}$$

show that the pion pole appears in $\mathcal{M}_1$ as

$$\mathcal{M}_1 = -i\frac{f_\pi}{q^2} A = i\frac{e^2}{4\pi^2}\frac{1}{q^2} \tag{7}$$

(d) Recalling a factor 1/2 in the phase space integration due to the identical photons, and the expression for the decay width into two particles given in Peskin et al, show that

$$\Gamma(\pi^0 \to 2\gamma) = \frac{\alpha^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2} = \frac{1}{0.77\cdot 10^{-16}\ \mathrm{s}}$$

which is in good agreement with $\tau_{exp} = (0.84 \pm .06)\cdot 10^{-16}$ s.

Note: You can solve a later task without having done the previous ones.

1 a) $\mathcal{L}^{(2)} = \frac{1}{4}F^2\,\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^\dagger + 2B(MU^\dagger + M^\dagger U)\right]$ QCD 12/08

$$= +\frac{1}{2}(\partial_\mu \pi^0)^2 - \frac{1}{2}m_{\pi^0}^2\,\pi^{0\,2} + \frac{1}{2}(\partial_\mu \eta)^2 - \frac{1}{2}m_\eta^2\,\eta^2 + \dots$$

$$M = \begin{pmatrix} m_u & & 0 \\ & m_d & \\ 0 & & m_s \end{pmatrix}, \quad U = \exp\left\{\frac{i}{F}\begin{pmatrix} \phi_3 + \phi_8/\sqrt{3} & \sqrt{2}\,\pi^+ & \sqrt{2}\,k^+ \\ \sqrt{2}\,\pi^- & -\phi_3 + \phi_8/\sqrt{3} & \sqrt{2}\,k^0 \\ \sqrt{2}\,k^- & \sqrt{2}\,\bar{k}^0 & -2\phi_8/\sqrt{3} \end{pmatrix}\right\}$$

Expanding U to second order, ϕ_3 and ϕ_8 appear on the diagonal of $MU^\dagger = M^\dagger U$ $(\phi_3 = \phi_3^\dagger,\ \phi_8 = \phi_8^\dagger)$:

$$\mathcal{L}^{(2)} = \frac{1}{4}F^2\,\frac{i^2}{2F^2}\cdot 2\cdot 2B\,\mathrm{Tr}\begin{pmatrix} m_u(\phi_3+\phi_8/\sqrt{3})^2 & \cdot & \cdot \\ \cdot & m_d(-\phi_3+\phi_8/\sqrt{3})^2 & \cdot \\ \cdot & \cdot & m_s\frac{4}{3}\phi_8^2 \end{pmatrix}$$

$$= -\frac{B}{2}\begin{pmatrix}\phi_3 & \phi_8\end{pmatrix}\begin{pmatrix} m_u + m_d & \frac{1}{\sqrt{3}}(m_u - m_d) \\ \frac{1}{\sqrt{3}}(m_u - m_d) & \frac{1}{3}(m_u + m_d + 4m_s) \end{pmatrix}\begin{pmatrix}\phi_3 \\ \phi_8\end{pmatrix}$$

$$= -\frac{B}{2}\left\{\phi_3^2(m_u+m_d) + \frac{2}{\sqrt{3}}(m_u-m_d)\phi_3\phi_8 + \frac{1}{3}(m_u+m_d+4m_s)\phi_8^2\right\}$$

1 b) Eigenvalues

$$\begin{vmatrix} m_u + m_d - \lambda & \frac{1}{\sqrt{3}}(m_u - m_d) \\ \frac{1}{\sqrt{3}}(m_u - m_d) & \frac{1}{3}(m_u + m_d + 4m_s) - \lambda \end{vmatrix} =$$

$$= \lambda^2 - \frac{4}{3}\lambda(m_u + m_d + m_s) + \frac{1}{3}(m_u+m_d)^2 + \frac{4}{3}m_s(m_u+m_d) - \frac{1}{3}(m_u-m_d)^2 = 0$$

16 cont.)

$$\lambda = \frac{2}{3}(m_u + m_d + m_s) \pm \frac{1}{3}\sqrt{4(m_u+m_d+m_s)^2 - 3(m_u+m_d)^2 - 12 m_s (m_u+m_d) + 3(m_u-m_d)^2}$$

$$= \frac{2}{3}(m_u + m_d + m_s) \pm \frac{1}{3}\sqrt{(m_u + m_d - 2m_s)^2 + 3(m_u - m_d)^2}$$

$$\lambda_1 = m_u + m_d + \mathcal{O}(m_u - m_d)^2$$

$$\lambda_2 = \frac{1}{3}(m_u + m_d) + \frac{4}{3} m_s + \mathcal{O}(m_u - m_d)^2$$

$$\Rightarrow \quad m_{\pi^0}^2 = B(m_u + m_d)$$

$$m_\eta^2 = \frac{B}{3}(m_u + m_d + 4m_s)$$

2a) $\mathcal{L}_q = \bar{\Psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi$

$$\frac{\delta}{\delta\bar{\Psi}} : \quad (i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi = 0 \qquad |\ \bar{\Psi}\gamma_5 \times$$

add

$$\frac{\delta}{\delta\Psi} : \quad \bar{\Psi}(-i\overleftarrow{\partial\!\!\!/} - gA\!\!\!/ - m) = 0 \qquad |\ \times \gamma_5\Psi$$

$$\bar{\Psi}(i\gamma_5\overrightarrow{\partial\!\!\!/} + i\gamma_5\overleftarrow{\partial\!\!\!/} - 2m\gamma_5)\Psi = 0$$

$$i\partial_\mu[\bar{\Psi}\gamma^\mu\gamma_5\Psi] = -2m\bar{\Psi}\gamma_5\Psi\ ; \qquad j_5^\mu \equiv \bar{\Psi}\gamma^\mu\gamma_5\Psi$$

$$\partial_\mu j_5^\mu = 2im\bar{\Psi}\gamma_5\Psi \qquad = 0 \text{ for } m = 0$$

2b) $\partial_\mu j_5^\mu = -\dfrac{e^2}{16\pi^2}\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$ $\qquad E^i = F^{0i}$, $\quad B^i = \frac{1}{2}\varepsilon^{ijk}F^{jk}$

$$j_5^\mu = \frac{1}{4}\underbrace{\bar{\Psi}(1-\gamma_5)}_{2\bar{\Psi}_R}\gamma^\mu\underbrace{(1+\gamma_5)\Psi}_{2\Psi_R} - \frac{1}{4}\underbrace{\bar{\Psi}(1+\gamma_5)}_{2\bar{\Psi}_L}\gamma^\mu\underbrace{(1-\gamma_5)\Psi}_{2\Psi_L} = \bar{\Psi}_R\gamma^\mu\Psi_R - \bar{\Psi}_L\gamma^\mu\Psi_L$$

$$\int d^4x\,\partial_\mu j_5^\mu = \int d^3\vec{x}\,[j_5^0(x^0 = +\infty, \vec{x}) - j_5^0(x^0 = -\infty, \vec{x})] = \Delta N_R - \Delta N_L$$

$N_R = \int d^3\vec{x}\,\Psi_R^+\Psi_R$ and we assumed $j_5^k(\vec{x} \to \infty) = 0$ $(k = 1,2,3)$

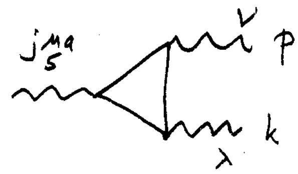
RHS of ABJ-anomaly eq: By symmetry, can set $\alpha = 0$ and multiply $\times 4$:

$$\partial_\mu j_5^\mu = \frac{e^2}{4\pi^2}\varepsilon^{ijk}F^{0i}F^{jk} = \frac{e^2}{2\pi^2}\vec{E}\cdot\vec{B}$$

$$\Rightarrow \Delta N_R - \Delta N_L = \frac{e^2}{2\pi^2}\int d^4x\,\vec{E}\cdot\vec{B}$$

3.

$\pi^0 \to \gamma\gamma$ EM anomaly from



$$\partial_\mu j_5^{\mu a} = -\frac{e^2}{16\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \operatorname{Tr}\left[\tau^a Q^2\right]$$

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \qquad \text{3 colors: Factor 3}$$

$$\operatorname{Tr}(\tau^3 Q^2) = \operatorname{Tr}\left[\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \frac{4}{9} & 0 \\ 0 & \frac{1}{9} \end{pmatrix}\right] = \frac{1}{2}\operatorname{Tr}\begin{pmatrix} \frac{4}{9} & \\ & -\frac{1}{9} \end{pmatrix} = \frac{1}{6}$$

$$\partial_\mu j_5^{\mu 3} = -\frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

Define: $\langle \gamma(p)\gamma(k) | j_5^{\mu 3}(q) | 0 \rangle = \varepsilon^*_\nu(p)\varepsilon^*_\lambda(k) M^{\mu\nu\lambda}(p,k) \qquad (q = p+k)$

a) Ward id: $p_\nu M^{\mu\nu\lambda}(p,k) = k_\lambda M^{\mu\nu\lambda} = 0$

b) Identical γ's: $M^{\mu\nu\lambda}(p,k) = M^{\mu\lambda\nu}(k,p)$

c) Parity: $M^{\mu\nu\lambda}(p,k) = \mp M^{\mu\nu\lambda}(k,p) \qquad - : \mu = 0, \quad + : \mu = 1,2,3$

The last requiremt follows from parity invariance for $1^+ \to 1^- 1^-$. In a frame where $\vec{q} = 0$, parity interchanges p and k.

$$M^{\mu\nu\lambda}(k,p) = q^\mu \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta M_1 + (\varepsilon^{\mu\nu\alpha\beta} k^\lambda - \varepsilon^{\mu\lambda\alpha\beta} p^\nu) k_\alpha p_\beta M_2$$
$$+ \left[(\varepsilon^{\mu\nu\alpha\beta} p^\lambda - \varepsilon^{\mu\lambda\alpha\beta} k^\nu) k_\alpha p_\beta - \varepsilon^{\mu\nu\lambda\sigma}(p-k)_\sigma \, p\cdot k\right] M_3$$

where $M_i = M_i(q^2)$.

$q^\mu \left(g^{\nu\lambda} - 2\frac{k^\nu p^\lambda}{q^2}\right)$ would satisfy a & b, but not c.

3a) Check of conditions:

a) $$p_\nu M^{\mu\nu\lambda} = q^\mu \varepsilon^{\nu\lambda\alpha\beta} \overset{0}{p_\nu p_\alpha} k_\beta M_1 + (\overset{0}{\varepsilon^{\mu\nu\alpha\beta} p_\nu} k^\lambda - \overset{0}{\varepsilon^{\mu\lambda\alpha\beta} p^2}) k_\alpha p_\beta M_2$$

$$+ [(\overset{0}{\varepsilon^{\mu\nu\alpha\beta} p_\nu} p^\lambda - \varepsilon^{\mu\lambda\alpha\beta} p.k) k_\alpha p_\beta - \varepsilon^{\mu\nu\lambda\sigma} p_\nu (\overset{0}{p_\sigma} - k_\sigma) p.k] M_3$$

$$= 0 \quad \text{ok}$$

b) $$M^{\mu\lambda\nu}(k,p) = q^\mu \varepsilon^{\lambda\nu\alpha\beta} k_\alpha p_\beta M_1 + (\varepsilon^{\mu\lambda\alpha\beta} p^\nu - \varepsilon^{\mu\nu\alpha\beta} k^\lambda) p_\alpha k_\beta M_2$$

$$+ [(\varepsilon^{\mu\lambda\alpha\beta} k^\nu - \varepsilon^{\mu\nu\alpha\beta} p^\lambda) p_\alpha k_\beta - \varepsilon^{\mu\lambda\nu\sigma} (k-p)_\sigma p.k] M_3$$

$$= M^{\mu\nu\lambda}(k,p) \quad \text{ok}$$

c) $$M^{\mu\nu\lambda}(k,p) = q^\mu \varepsilon^{\nu\lambda\alpha\beta} k_\alpha p_\beta M_1 + (\varepsilon^{\mu\nu\alpha\beta} p^\lambda - \varepsilon^{\mu\lambda\alpha\beta} k^\nu) p_\alpha k_\beta M_2$$

$$+ [(\varepsilon^{\mu\nu\alpha\beta} k^\lambda - \varepsilon^{\mu\lambda\alpha\beta} p^\nu) p_\alpha k_\beta - \varepsilon^{\mu\nu\lambda\sigma} (k-p)_\sigma p.k] M_3$$

In $\vec{q}=0$ frame: $\vec{k} = -\vec{p}$

$\mu = 0$: $M_1 \to -M_1$ ok

$M_2 \to 0$ since $\alpha, \beta = 1,2,3$ & $\vec{p} \times \vec{k} = 0$

$M_3 \to -M_3$ ok

$\mu = 1,2,3$: $M_1 \to 0$

$M_2 \to +M_2$ since $\lambda, \nu = 1,2,3$ & sign changes from $k_\alpha p_\beta \to p_\alpha k_\beta$ and $k^\lambda \to p^\lambda$, $p^\nu \to k^\nu$

$M_3 \to +M_3$ since last term vanishes: $\sigma = 1,2,3$ and so either $\nu = 0$ or $\lambda = 0$ & can choose $\varepsilon^0(p) = \varepsilon^0(k) = 0$

3b)

$$i q_\mu M^{\mu\nu\lambda}(p,k) = i q^2 \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta M_1 - i\varepsilon^{\mu\nu\lambda\sigma} q_\mu (p-k)_\sigma\, p\cdot k\, M_3$$

$$\left.\begin{array}{l} q = p+k \\ q^2 = 2p\cdot k \end{array}\right| \qquad = i q^2 \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta (M_1 + M_3)$$

$$\int d^4x\, e^{-iq\cdot x} \langle \gamma(p)\gamma(k) | \partial_\rho j_5^\rho(x) | 0\rangle = i q_\rho M^{\rho\sigma\tau} \varepsilon^*_\sigma(p) \varepsilon^*_\tau(k)$$

$$= -\frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu} \qquad \langle 0 | a(p,\lambda_p)\, a(k,\lambda_k) \int d^4x\, e^{-iq\cdot x}$$

$$* \left[\partial_\alpha A_\beta(x) - \partial_\beta A_\alpha(x)\right]\left[\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)\right] |0\rangle$$

$$A_\beta(x) = \int \frac{d^3 q_1}{(2\pi)^3 E_1} \left[a(q_1)\, \varepsilon_\beta(q_1) e^{-iq_1\cdot x} + a^+(q_1)\, \varepsilon^*_\beta(q_1) e^{iq_1\cdot x}\right]$$

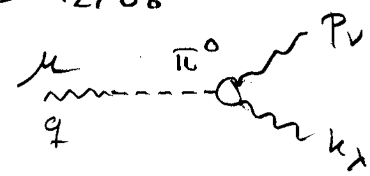
$$\left[a(p,\lambda), a^+(q_1,\lambda_1)\right] = (2\pi)^3 2E_1\, \delta^3(\vec{p} - \vec{q}_1)\, \delta_{\lambda,\lambda_1}$$

$$i q_\rho M^{\rho\sigma\tau} \varepsilon^*_\sigma(p) \varepsilon^*_\tau(k) = -\frac{e^2}{32\pi^2} \varepsilon^{\alpha\beta\mu\nu}\, 4\cdot 2\, (i p_\alpha)(i k_\mu)$$

$$* \varepsilon^*_\beta(p)\, \varepsilon^*_\nu(k) \left[(2\pi^4)\, \delta^4(q-p-k)\right]$$

$$\Rightarrow i q_\mu M^{\mu\nu\lambda} = +\frac{e^2}{4\pi^2} \varepsilon^{\alpha\nu\mu\lambda} p_\alpha k_\mu = -\frac{e^2}{4\pi^2} \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta$$

3c) QCD 12/08



$j_5^{\mu 3}$ can couple to a π^0,
giving a $\frac{1}{q^2}$ pole in $M_1 + M_3$.

Let $iM(\pi^0 \to 2\gamma) = i A\, \varepsilon_\nu^*(p)\, \varepsilon_\lambda^*(k)\, \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta$

Check: 1) $p_\nu \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta = p_\lambda \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta = 0$

2) Symmetry unde $p \leftrightarrow k$,

3) Change of sign for $\vec{p} \to -\vec{p}$, $\vec{k} \to -\vec{k}$
ok since ε's dont change (axial vectors)
and with $\varepsilon^0 = 0$ either α or $\beta = 1, 2, 3$.

Contribution of π^0 pole to $M^{\mu\nu\lambda}$:

$$\langle \gamma(p)\gamma(k) | \pi^0 \rangle \langle \pi^0 | j_5^{\mu 3} | 0 \rangle = i q^\mu f_\pi \frac{i}{q^2} i A\, \varepsilon_\nu^*(p)\, \varepsilon_\lambda^*(k)$$
$$* \, \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta = \varepsilon_\nu^*(p)\, \varepsilon_\lambda^*(k)\, M_\pi^{\mu\nu\lambda}(p,k)$$

$$M_\pi^{\mu\nu\lambda}(p,k) = -\frac{i}{q^2} q^\mu f_\pi A\, \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta$$

i.e. $M_1^\pi = -\frac{i}{q^2} f_\pi A + \mathcal{O}((q^2)^0)$

Comparing the expressions for $q_\mu M^{\mu\nu\lambda}$:

$$i q^2 M_1^\pi = -\frac{e^2}{4\pi^2} \Rightarrow A = -\frac{e^2}{4\pi^2}\frac{1}{f_\pi} = -\frac{\alpha}{\pi}\frac{1}{f_\pi}$$

3d) QCD 12/08

$$\Gamma(\pi^0 \to 2\gamma) = \frac{1}{2m_\pi} \underbrace{\frac{1}{2}}_{\text{i'd. particles}} \frac{1}{32\pi^2} \underbrace{\frac{2p_{CM}}{E_{CM}}}_{=1} \int d\Omega \, |A|^2 \sum_{\lambda_p \lambda_k} |\varepsilon^*_\nu(p)\, \varepsilon^*_\lambda(k)$$
$$* \, \varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta|^2$$

$$\sum_\lambda \varepsilon^*_\nu(p)\, \varepsilon_{\nu'}(p) \longrightarrow -g_{\nu\nu'}$$

$$\varepsilon^{\nu\lambda\alpha\beta}\, \varepsilon^{\nu'\lambda'\alpha'\beta'} (-g_{\nu\nu'})(-g_{\lambda\lambda'}) = \varepsilon^{\nu\lambda\alpha\beta}\, \varepsilon_{\nu\lambda}{}^{\alpha'\beta'}$$

$$\varepsilon^{\nu\lambda\alpha\beta} p_\alpha k_\beta = \varepsilon^{\nu\lambda\alpha\beta} p_\alpha (p_\beta + k_\beta) = m_\pi\, \varepsilon^{ijk} p_k$$

$$\varepsilon^{ijk}\, \varepsilon_{ij}{}^{k'} = 2\delta_{k,k'} \quad ; \quad \vec{p}^{\,2} = \frac{1}{4} m_\pi^2$$

$$\Gamma(\pi^0 \to 2\gamma) = \frac{|A|^2}{32\pi^2 \cdot 4m_\pi} 4\pi \, \frac{2}{4} m_\pi^2 \cdot m_\pi^2 = \frac{|A|^2}{64\pi} m_\pi^3 = \frac{\alpha^2}{64\pi^3} \frac{m_\pi^3}{f_\pi^2}$$

$$\tau(\pi^0 \to 2\gamma) = 64\pi^3 (137)^2 \left(\frac{.093}{.14}\right)^2 \frac{6.582 \cdot 10^{-25}}{.14\ \text{GeV}} \ \text{GeV} \cdot \text{s}$$

$$= 0.77 \cdot 10^{-16}\ \text{s} \qquad (\exp = (.84 \pm .06) \cdot 10^{-16}\ \text{s}$$