Session of Monday 15 September at 16-18 in aud A315.

1. The dependence of the QCD coupling $\alpha_s(\mu^2)$ on the renormalization scale μ^2 is to lowest order given by
$$\mu^2 \frac{d\alpha_s}{d\mu^2} = -\beta_0 \alpha_s^2; \qquad \beta_0 = \frac{33 - 2n_f}{12\pi} \tag{1}$$
where n_f is the number of active flavors.

   (a) Show that $\alpha_s(\mu^2) = 1/\beta_0 \ln(\mu^2/\Lambda_{QCD}^2)$, where Λ_{QCD} is a constant of integration.

   (b) Determine Λ_{QCD} by fixing $\alpha_s(M_Z^2) = 0.119$ assuming $n_f = 3$. What is then the value of α_s at $\mu = 3$ GeV?

   (c) Assume that the coupling "freezes" at low μ^2 by the replacement $\mu^2 \to \mu^2 + m_\rho^2$. What is then the value of $\alpha_s(0)$?

2. Bound state wave functions are defined at an equal time t of the constituents, or alternatively at equal Light-Front time $x^+ \equiv t + z$. This implies the use of time- (or LF time-)ordered perturbation theory.

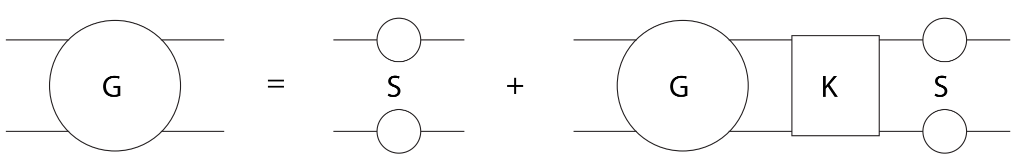
   (a) Evaluate the time-ordered propagators
$$\begin{aligned} S(t, \boldsymbol{p}) &= \int \frac{dp^0}{2\pi} S(p) \exp(-itp^0) \\ S(x^+, \boldsymbol{p}) &= \int \frac{dp^-}{2\pi} S(p) \exp(-ix^+ p^-/2) \end{aligned}$$
   where $p^- \equiv p^0 - p^3$ and $S(p) = i/(p^2 - m^2 + i\varepsilon)$ is a free scalar propagator.

   (b) In ϕ^3 theory the Feynman amplitude for $a + b \to c + d$ with an s-channel particle of mass M is $\mathcal{M} = -g^2/[(p_a + p_b)^2 - M^2 + i\varepsilon]$. Find this amplitude using time-ordered perturbation theory, by fixing the abM vertex to be at $t = 0$ and integrating over the time t of the Mcd vertex. *Note:* The wave functions of the outgoing particles provide a phase factor $\exp[i(E_c + E_d)t]$.

   (c) Do the previous calculation using x^+-ordered perturbation theory. Compare and discuss the various methods of evaluating the amplitude.

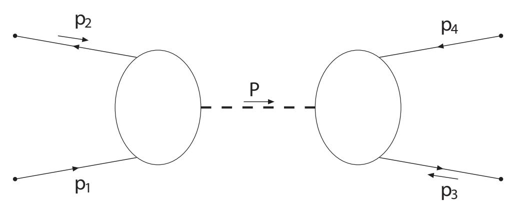
3. Discuss the calculation of bound states in field theory, at the level of Feynman graphs.

   (a) In the Dyson-Schwinger equation

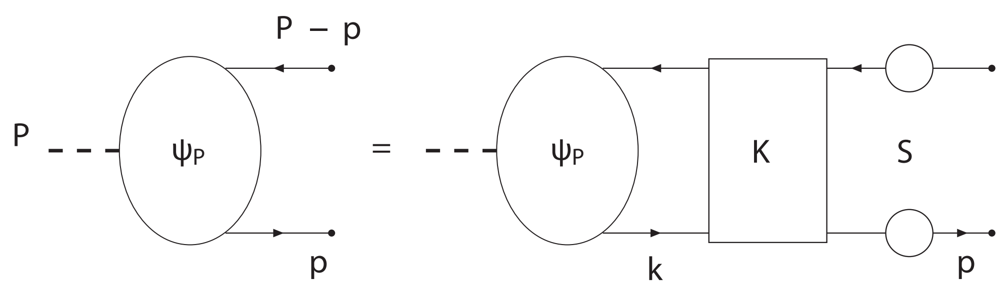


the kernel K is the sum of all 2-particle irreducible graphs, with its external propagators amputated. Verify which class of Feynman diagrams in the Green function G appear once on both the right- and left-hand sides when the one-particle propagators S are the sum of all connected 2-point functions.

(b) A bound-state appears as a pole in G, with a residue which factorizes into wave functions for the in- and out-going states:



Verify that this implies that the wave function satisfies the Bethe-Salpeter equation:



(c) The non-relativistic Hydrogen atom is obtained in this framework by using free propagators and a single photon exchange kernel. Show which diagrams generate the pole in the electron-proton scattering amplitude. How can diagrams with many photon exchanges be important, given that $\alpha \simeq 1/137$ is small?

4. If time allows, continue the discussion of facilities for QCD and hadron physics.