Session of Monday 22 September at 16-17.30 in aud A315.

1. Consider the kinematics of electron scattering on nucleons, $e(k) + N(p) \to e(k') + X(p')$ at large $s = (k+p)^2$. The initial momenta are $k \equiv (k^+, k^-, \boldsymbol{k}_\perp) = (0, k^-, \boldsymbol{0}_\perp)$ and $p = (m, m, \boldsymbol{0}_\perp)$, where $k^\pm = k^0 \pm k^3$, and the virtual photon carries $q = k - k'$.

   (a) For elastic scattering $e(k) + N(p) \to e(k') + N(p')$, determine the maximal value of $|\boldsymbol{k'}_\perp|$, neglecting terms of $\mathcal{O}\left(m^2/s\right)$.
   Hint: Consider the momenta in the CM frame.

   (b) For this value of $|\boldsymbol{k'}_\perp|$ in elastic scattering, determine the photon virtuality $Q^2 = -q^2 = -(k-k')^2$, the photon energy $\nu = q^0$ in the nucleon rest frame and the Bjorken variable $x_B \equiv Q^2/2m\nu$.

   (c) Assuming that the electron scatters on a quark in the nucleon which has momentum $p_q = (xp^+, 0, \boldsymbol{0}_\perp)$, express x in terms of x_B and determine the squared invariant mass $W^2 = (k + p - k')^2$ of the final hadronic system in terms of x_B and Q^2.

2. Assuming (for simplicity) the electron and nucleon to be scalar (spin 0) particles, the $e(k) + N(p) \to e(k') + X(p')$ scattering amplitude is $A = e^2(k+k')_\mu j_X^\mu(p,p')/q^2$. For an elementary (pointlike) nucleon we would have $j_N^\mu = (p+p')^\mu$.

   (a) If the charge of the nucleon were smoothly distributed its current might be described by the Fourier transform of a gaussian distribution in position $\boldsymbol{x}$,

   $$j_N^\mu(p,p') = (p+p')^\mu \int d^4x\, e^{iq\cdot x} \exp\left(-\frac{\boldsymbol{x}^2}{R^2}\right)$$

   Show that the scattering amplitude then vanishes exponentially as $Q^2 \to \infty$.

   (b) The inclusive cross section $\sum_X \sigma(eN \to eX)$ is proportional to $e^4 L^{\mu\nu} W_{\mu\nu}/Q^4$, where the lepton vertex $L^{\mu\nu}(k,q) = (k+k')^\mu (k+k')^\nu$ and $W^{\mu\nu}(p,q)$ describes the hadronic vertex. Use Lorentz invariance to express $W^{\mu\nu} = W^{\nu\mu}$ in terms of two structure functions $W_1(x_B, Q^2)$ and $W_2(x_B, Q^2)$ multiplied by Lorentz tensors which ensure the gauge invariance condition $q_\mu W^{\mu\nu}(p,q) = 0$. Here $x_B = Q^2/2p\cdot q$.

   (c) Find the contribution to W_1 and W_2 from scattering on a scalar quark of charge ee_q and momentum $p_q = xp$. Compare the Q^2-dependence of $W_{1,2}$ with that given by the squared nucleon current in (a).

3. Consider $e(p) + \mu(P) \to e(p') + \gamma(k) + \mu(P')$ as a model of gluon emission in hard scattering. Assume the electron and muon to be massless and spinless. Denote $p = (p^+, 0, \boldsymbol{0}_\perp)$, $P = (0, P^-, \boldsymbol{0}_\perp)$, $k = (xp^+, k_\perp^2/xp^+, \boldsymbol{k}_\perp)$, $q = P - P'$ and $s = (p+P)^2$. Let $p^+ \to \infty$ keeping P^-, x, $\boldsymbol{k}_\perp$ and $\boldsymbol{q}_\perp$ fixed.

(a) Express q^+ and q^- in terms of the variables mentioned above and the invariant mass $M^2 = (k+p')^2$ of the produced γe system. Show that $q^2 \simeq -q_\perp^2$.

(b) Show that

$$M^2 = \frac{(\boldsymbol{k}_\perp - x\boldsymbol{q}_\perp)^2}{x(1-x)}$$

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(c) Use the photon polarization vector $\epsilon_\lambda(k)$ given in Eq. (5) of the "LF Spinors and Polarization Vectors" file on the home page to derive the expression for the Feynman amplitude where the photon is emitted from the initial (A_a) and final (A_b) electron:

$$\begin{aligned} A_a &= \frac{4e^3}{q_\perp^2} \frac{(1-x)s}{k_\perp^2} \boldsymbol{e}_\lambda \cdot \boldsymbol{k}_\perp \\ A_b &= -\frac{4e^3}{q_\perp^2} \frac{(1-x)s}{(\boldsymbol{k}_\perp - x\boldsymbol{q}_\perp)^2} \boldsymbol{e}_\lambda \cdot (\boldsymbol{k}_\perp - x\boldsymbol{q}_\perp) \end{aligned} \tag{1}$$