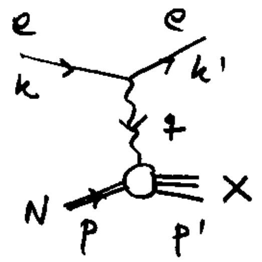
1. $eN \to eX$



$k = (0, k^-, \vec{0}_\perp)$

$p = (m, m, \vec{0}_\perp)$

$s = (k+p)^2 = m^2 + mk^- \simeq mk^-$ $\qquad q = k - k'$

a) $X = N$. In CM : $k_{CM} \simeq (0, \sqrt{s}, \vec{0}_\perp)$

$p_{CM} \simeq (\sqrt{s}, 0, \vec{0}_\perp)$

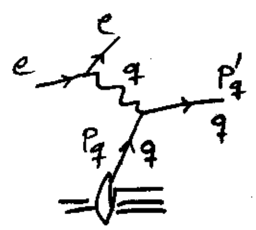
$\Rightarrow |\vec{k}'_\perp| \leq |\vec{k}_{CM}| = \frac{1}{2}\sqrt{s}$ $\qquad \left(\Rightarrow k'^{z}_{CM} = 0 ;\ k'^{\pm}_{CM} = \frac{1}{2}\sqrt{s}\right)$

b) $Q^2 = -(k-k')^2 = 2\,k \cdot k' = k^-_{CM}\, k'^+_{CM} = \sqrt{s} \cdot \frac{1}{2}\sqrt{s} = \frac{1}{2} s$

$\nu = q^0_{Lab} = \frac{1}{m}(k-k') \cdot p = \frac{1}{2m}(k-k')^-_{CM}\, p^+_{CM} = \frac{1}{2m}\,\frac{1}{2}\sqrt{s} \cdot \sqrt{s} = \frac{s}{4m}$

$x_B = \frac{Q^2}{2m\nu} = \frac{\frac{1}{2}s}{2m \cdot s/4m} = 1$

c) $p_q = (x p^+, 0, \vec{0}_\perp)$ $\Rightarrow m_q = 0$



$p'^2_q = (k - k' + p_q)^2 = -Q^2 + (k-k')^-_{CM} \times p^+_{CM} = 0$

$x = \frac{Q^2}{2\, q \cdot p} = x_B$

$W^2 = (k - k' + p)^2 \simeq -Q^2 + 2 q \cdot p = Q^2\left(\frac{1}{x} - 1\right) = \frac{1-x_B}{x_B} Q^2$

2. $A(eN \to eX) = e^2 (k+k')^\mu j_\mu(p,p')/q^2$ for given X,

where j_μ depends on X.

a) $j^\mu = (p+p')^\mu \int d^4x\, e^{iq\cdot x} e^{-\vec{x}^2/R^2} = 2\pi\,\delta(q^0)(p+p')^\mu I$

$$I = \int d^3\vec{x}\, \exp\left[-\frac{1}{R^2}\left(\vec{x} + \frac{i}{2}R^2\vec{q}\right)^2 - \frac{1}{4}R^2\vec{q}^{\,2}\right]$$

$$= R^3 e^{-\frac{1}{4}R^2\vec{q}^{\,2}} \int d^3\left(\frac{\vec{x}}{R}\right) \exp\left[-\left(\frac{\vec{x}}{R} + \frac{i}{2}R\vec{q}\right)^2\right]$$

$\propto \exp\left[-\frac{1}{4}R^2Q^2\right]$, since $Q^2 = -(q^0)^2 + \vec{q}^{\,2} = \vec{q}^{\,2}$

b) $\sum_X \sigma(eN \to e'X) \propto \dfrac{e^4}{Q^4} L^{\mu\nu} W_{\mu\nu}$;

$L^{\mu\nu}(k,q) = (k+k')^\mu (k+k')^\nu$; $\quad k' = k - q$

$q_\mu L^{\mu\nu}(k,q) = \left[(k-k')\cdot(k+k')\right](k+k')^\nu = 0$ ok

$$W^{\mu\nu}(p,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1(x_B, Q^2) + W_2 \frac{1}{m^2} \underbrace{\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right)}_{p^\mu + \frac{1}{2x_B} q^\mu} \left(p^\nu - \frac{p\cdot q}{q^2} q^\nu\right)$$

for which $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$.

c) $j_q^\mu = e_q(x_B p + x_B p + q)^\mu = e_q 2x_B \left(p^\mu + \dfrac{1}{2x_B} q^\mu\right)$

$\Rightarrow W_1 = 0$

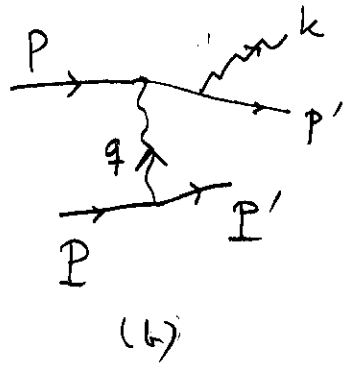
$W_2 = e_q^2 (2m x_B)^2 \delta(x - x_B)$ $\quad$ Indep. of Q^2

3. $p = (p^+, 0, \vec{0}_\perp)$

$P = (0, P^-, \vec{0}_\perp)$

$k = (x p^+, \frac{k_\perp^2}{x p^+}, \vec{k}_\perp)$

$p' = ((1-x) p^+ + q^+, q^- - \frac{k_\perp^2}{x p^+}, \vec{q}_\perp - \vec{k}_\perp)$



a) $P'^2 = (P-q)^2 = -2P\cdot q + q^2 = -q^+P^- + q^2 = 0; \quad q^+ = \frac{q^2}{P^-}$

$(k+p')^2 = M^2 = (q+p)^2 = q^2 + p^+q^- ; \quad q^- = \frac{M^2 - q^2}{p^+}$

Since $M^2 = k^+ p'^- + k^- p'^+ - 2\vec{k}_\perp \cdot \vec{p}'_\perp$ is fixed as $p^+ \to \infty$

we have $q^- \to 0$, hence $q^2 = q^+ q^- - q_\perp^2 \simeq -q_\perp^2$

b) $M^2 = x p^+ (q^- - \frac{k_\perp^2}{x p^+}) + \frac{k_\perp^2}{x p^+}(1-x) p^+ - 2\vec{k}_\perp \cdot (\vec{q}_\perp - \vec{k}_\perp)$

$= x(M^2 + q_\perp^2) - k_\perp^2 + \frac{1-x}{x} k_\perp^2 - 2\vec{k}_\perp \cdot (\vec{q}_\perp - \vec{k}_\perp)$

$$M^2 = \frac{k_\perp^2}{x(1-x)} + \frac{\overbrace{x}^{1-(1-x)} q_\perp^2 - 2\vec{q}_\perp \cdot \vec{k}_\perp}{1-x} = \frac{k_\perp^2 + x(\vec{q}_\perp - \vec{k}_\perp)^2 - x k_\perp^2}{x(1-x)} - q_\perp^2$$

$$= \frac{k_\perp^2}{x} + \frac{(\vec{q}_\perp - \vec{k}_\perp)^2}{1-x} - q_\perp^2$$

$$= \frac{k_\perp^2 + x^2 q_\perp^2 - 2x\, \vec{q}_\perp \cdot \vec{k}_\perp}{x(1-x)} = \frac{(\vec{k}_\perp - x\vec{q}_\perp)^2}{x(1-x)}$$

3c) $\varepsilon_\lambda(k) = \left(0, \frac{2\vec{e}_\lambda \cdot \vec{k}_\perp}{k^+}, \vec{e}_\lambda\right)$; $\vec{e}_\lambda = \frac{-1}{\sqrt{2}}(\lambda, i)$

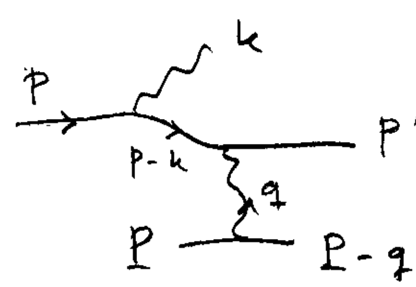
satisfies $k \cdot \varepsilon(k) = 0$ ok ;

$$i A_b = (-ie)\, \varepsilon_\lambda(k) \cdot (\overset{p+q-\overset{\circ}{k}}{\widetilde{p'}} + p + q) \frac{i}{(p'+k)^2} (-ie)^2 \underbrace{(p+p'+k)\cdot(P + P - \overset{\circ}{q})}_{P^-(p+p'+k)^+ = 2p^+P^- = 2s} \frac{-i}{q^2}$$

$$s \equiv (p+P)^2 = 2p\cdot P = p^+ P^-$$

$$2\varepsilon_\lambda(k)\cdot(p+q) = \varepsilon_\lambda^-(p^+ + \overset{\circ}{q^+}) - 2\vec{e}_\lambda \cdot \vec{q}_\perp = 2\vec{e}_\lambda \cdot \left(\frac{\vec{k}_\perp}{x} - \vec{q}_\perp\right)$$

$$A_b = -\frac{4e^3}{q_\perp^2} \frac{s}{xM^2} \vec{e}_\lambda \cdot (\vec{k}_\perp - x\vec{q}_\perp)$$



(a)

$$(p-k)^2 = -2p\cdot k = -p^+k^- = -\frac{k_\perp^2}{x}$$

$$i A_a = (-ie)^2 (p' + p - k)\cdot(P + P - \overset{\circ}{q}) \frac{-i}{q^2} \frac{i}{(p-k)^2} (-ie)\, \varepsilon_\lambda(k)\cdot(2p - \overset{\circ}{k})$$

$$A_a = \frac{4e^3}{q_\perp^2} \frac{(1-x)s}{k_\perp^2/x} \vec{e}_\lambda \cdot \frac{\vec{k}_\perp}{x} = \frac{4e^3}{q_\perp^2} \frac{(1-x)s}{k_\perp^2} \vec{e}_\lambda \cdot \vec{k}_\perp$$

$$A_b = -\frac{4e^3}{q_\perp^2} \frac{(1-x)s}{(\vec{k}_\perp - x\vec{q}_\perp)^2} \vec{e}_\lambda \cdot (\vec{k}_\perp - x\vec{q}_\perp)$$