

Session of Monday 6 October at 16.00-17.30 in aud A315.

1. (a) Prove the following identity for a fermion propagator:

$$\frac{\not{p} + m}{p^2 - m^2 + i\varepsilon} = \frac{1}{2E_p} \sum_s \left[ \frac{u(\mathbf{p}, s)\bar{u}(\mathbf{p}, s)}{p^0 - E_p + i\varepsilon} + \frac{v(-\mathbf{p}, s)\bar{v}(-\mathbf{p}, s)}{p^0 + E_p - i\varepsilon} \right]$$

where  $E_p = +\sqrt{\mathbf{p}^2 + m^2}$ . Note that the spinors are independent of  $p^0$ .

Do a Fourier transform of the propagator from  $p^0$  to time  $t$ , and give an interpretation of the two terms on the *rhs*.

- (b) Find the corresponding identity in terms of the LF spinors defined in eqs. (14) - (15) of the ‘‘LF Spinors and Polarization Vectors’’ file on the home page, where the ‘-’ momentum in the spinors is defined to be  $p_{LF}^- \equiv (\mathbf{p}_\perp^2 + m^2)/p^+$ .

Do a Fourier transform from  $p^-$  to  $x^+$  and give your interpretation.

2. (a) Calculate the amplitude for a massless electron of momentum  $p = (p^+, 0^-, \mathbf{0}_\perp)$  and positive helicity to radiate a photon with momentum  $k = (xp^+, k_\perp^2/xp^+, \mathbf{k}_\perp)$  and helicity  $\lambda = \pm 1$ . You may use the LF spinors and polarization vectors given on the home page.

- (b) Discuss the dependence of the photon polarization on  $x$ . Find the relation between the squared amplitude and the splitting function  $P_{\gamma/e}(x) = [1 + (1 - x)^2]/x$ .

3. Protons may emit soft photons coherently, *i.e.*, photons with a wavelength longer than the proton radius, which couple to the total charge of the proton.

- (a) Estimate the range in transverse momentum and energy of photons coherently emitted by the 7 TeV protons in the LHC. Are they interesting for the LHC experiments?

- (b) Evaluate the off-shellness  $p^2 - m^2$  of the emitting proton in the process  $N(p) \rightarrow N(p') + \gamma(k)$ . For a given  $x = k^+/p^+$ , estimate the range of  $\mathbf{k}_\perp$  in which the off-shellness is independent of the transverse momentum of the photon. From this, determine the effective radius of the photon cloud as a function of  $x$ . What is this radius (in fm) at the LHC for photons of 100 GeV?