Session of Monday 13 October at 16.00-17.30 in aud A315.

1. Derive the expression for the *equivalent photon approximation* (also known as the *Weizsäcker-Williams approximation*) of photon emission.

   (a) Using the results of Ex. 5, problems 2 and 3b, write the square of the amplitude for a virtual electron of momentum $p = (p^+, p^2/p^+, \mathbf{0})$ to emit a photon with $k = (xp^+, \boldsymbol{k}^2/k^+, \boldsymbol{k}_\perp)$. Assume $p^+ \gg \sqrt{p^2}$ so that you may keep only $p^- = p^-_{LF}$ pole part of the electron propagator. You should sum over the photon polarization and keep the electron mass m only in the virtual electron propagator.

   (b) Write the differential emission probability in x and $k_\perp$ by including the phase space factor.

   (c) Assuming that the virtual electron emerges from a process with hardness scale Q, integrate over $k_\perp$ to find the differential emission probability as a function of x:

$$P(e \to e\gamma) = \frac{\alpha}{2\pi} \log\left[\frac{Q^2}{(xm)^2}\right] \int dx \, \frac{1 + (1-x)^2}{x} \tag{1}$$

2. Let the momentum distribution of the electron from the hard process of 1(c) be $f_e(z, Q^2)$, where $z = p^+/P^+$ is the momentum fraction of some initial momentum P.

   (a) Based on the result (1), show that the Q^2-dependence of the electron momentum distribution is

$$\frac{df_e(z, Q^2)}{d\log Q^2} = \frac{\alpha}{2\pi} \int_z^1 \frac{dy}{y} \left[\frac{1+y^2}{(1-y)_+} + A\delta(1-y)\right] f_e(\frac{z}{y}, Q^2) \tag{2}$$

   Here the splitting function has been regularized by the '+' prescription: $\int dx f(x)/(1-x)_+ \equiv \int dx[f(x) - f(1)]/(1-x)$ and a δ-function contribution was added to account for the the virtual photon loop contribution.

   (b) Determine A in (2) from the electron number constraint, $\int dz f_e(z, Q^2) = 1$.

3. Apply the above result to $q \to q + g$ splitting in QCD.

   (a) Evaluate the color factor of this process, by summing over the final and averaging over the initial colors.

   (b) Determine the Q^2-dependence of the moments $M_n(Q^2) \equiv \int_0^1 dz \, z^{n-1} f_q(z, Q^2)$.

   (c) Show that $f_q(z, Q^2)$ decreases (increases) with Q^2 at high (low) z.