Session of Monday 20 October at 16.00-17.30 in aud A315.

1. The elastic form factors $F_{1,2}(Q^2)$ of the proton are defined in terms of the photon current matrix elements $F^\mu_{\lambda,\lambda'}$ as

$$F^\mu_{\lambda,\lambda'} \equiv \langle p+q/2, \lambda'|J^\mu(0)|p-q/2,\lambda\rangle = \bar{u}(p+q/2,\lambda')\left[F_1(Q^2)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(Q^2)\right]u(p-q/2,\lambda)$$

where $Q^2 = -q^2$, λ, λ' are the proton helicities and m is the proton mass. The photon current (for a given quark q) is $J^\mu(x) = \bar{\mathrm{q}}(x)\gamma^\mu \mathrm{q}(x)$, where $\mathrm{q}(x)$ is the quark field operator.

   (a) Show that $J^+(x) = \mathrm{q}_+^\dagger(x)\mathrm{q}_+(x)$ and is thus a density operator of the quark field $\mathrm{q}_+(x) \equiv \gamma^-\gamma^+\mathrm{q}(x)/4$, as stated in the lecture of Vanderhaeghen I.41.

   (b) Evaluate $F^+_{\lambda,\lambda'}$ in the frame where $q^+ \equiv q^0 + q^3 = 0$ and $\boldsymbol{p}_\perp = 0$, using the LF spinors on the home page. Verify that only $F_1(Q^2)$ contributes to $F^+_{\frac{1}{2},\frac{1}{2}}$.

   (c) Express the states $|p \pm q/2, s_\perp = \frac{1}{2}\rangle$, where the proton spin is orthogonal to to its momentum, in terms of the helicity states, and evaluate $\langle p+q/2, s_\perp = \frac{1}{2}|J^\mu(0)|p-q/2, s_\perp = \frac{1}{2}\rangle$. Compare with the expression for the transverse quark density on Vdh I.42.

2. The Generalized Parton Distributions H and E are defined by Vdh II.4 as

$$\frac{p^+}{2\pi}\int_{-\infty}^{\infty} dy^- e^{ixp^+y^-}\langle p+\tfrac{1}{2}\Delta, \lambda'|\bar{\mathrm{q}}(-\tfrac{1}{2}y)\gamma^+\mathrm{q}(\tfrac{1}{2}y)|p-\tfrac{1}{2}\Delta,\lambda\rangle_{y^+=y_\perp=0}$$
$$= \bar{u}(p+\tfrac{1}{2}\Delta,\lambda')\left[H(x,\xi,t)\gamma^+ + E(x,\xi,t)\frac{i\sigma^{+,\nu}\Delta_\nu}{2m}\right]u(p-\tfrac{1}{2}\Delta,\lambda)$$

where $\Delta^+ = -2\xi p^+$ and $t = \Delta^2$. Prove the relations on Vdh II.5:

$$\int_{-1}^{1} dx\, H(x,\xi,t) = F_1(t) \qquad \text{and} \qquad \int_{-1}^{1} dx\, E(x,\xi,t) = F_2(t)$$

Hint: The matrix element on the *lhs.*, as well as H and E, vanish for $|x| > 1$.

3. The Single Spin Asymmetry (SSA) of a scattering cross section $\sigma(s_a) \equiv \sigma(\vec{a} + b \to c + d)$ is defined as $A_{s_a} \equiv [\sigma(\vec{s}_a) - \sigma(-\vec{s}_a)]/[\sigma(\vec{s}_a) + \sigma(-\vec{s}_a)]$. Here $\vec{a}$ is a $j = \frac{1}{2}$ particle polarized in the direction $\vec{s}_a$. The spins of particles b, c and d are summed over. Particle a moves along the positive z-axis and the scattering angle $\theta_{ac} \neq 0, \pi$.

   (a) Let $T(\lambda_a, \lambda_b, \lambda_c, \lambda_d)$ be the scattering amplitude in the helicity basis ($\lambda_a = \pm\frac{1}{2}$ for $\vec{s}_a = \pm\hat{z} = (0,0,\pm 1)$). Show that parity invariance implies $\sigma(\lambda_a = \frac{1}{2}) = \sigma(\lambda_a = -\frac{1}{2})$, *i.e.*, $A_z = 0$. (*Hint:* Helicity transforms as you would expect classically.)

   (b) Derive the expression for the asymmetry when $\vec{s}_a = \hat{y} = (0,1,0)$,

$$A_y = \frac{2\,\mathrm{Im}\sum_{\lambda_b,\lambda_c,\lambda_d} T_+T_-^*}{\sum_{\lambda_b,\lambda_c,\lambda_d}(|T_+|^2 + |T_-|^2)} \quad \text{where} \quad T_\pm \equiv T(\lambda_a = \pm\tfrac{1}{2}, \lambda_b, \lambda_c, \lambda_d).$$

   Thus $A_y \neq 0$ requires both spin flip and a dynamical phase.