Session of Monday 27 October at 16.00-17.30 in aud A315.

1. Prove the "Gordon identity" on Vdh II.19:

$$\bar{u}(p+\tfrac{1}{2}\Delta,\lambda')\left[2m\gamma^{(\mu}p^{\nu)} - 2p^\mu p^\nu - p^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha\right]u(p-\tfrac{1}{2}\Delta,\lambda) = 0$$

where $(\mu,\nu) \equiv \frac{1}{2}(\mu\nu+\nu\mu)$ denotes symmetrization.

2. (a) Consider qualitatively, using the uncertainty relation, the ordinary (x^0) time development of a $\gamma^*(\nu,Q^2) \to q(z\nu,\boldsymbol{p}_\perp)\bar{q}((1-z)\nu,-\boldsymbol{p}_\perp)$ fluctuation in DIS. The photon energy ν is large and the quark energy is $E_q = \sqrt{(z\nu)^2+\boldsymbol{p}_\perp^2+m_q^2}$. Estimate the life-time of the fluctuation from the energy difference between the γ^* and $q\bar{q}$ states. Use the transverse velocity $\boldsymbol{v}_\perp = \boldsymbol{p}_\perp/E_q$ to estimate the transverse distance to which the quark and antiquark can separate during the life-time of the fluctuation. How does this distance depend on the magnitudes of $p_\perp$, m_q and z?

   (b) At the ep collider HERA electrons of 30 GeV collided with 800 GeV protons. Estimate the longitudinal distance, in the proton rest frame, between the $e\gamma^*e$ and $p\gamma^*X$ vertices for DIS events with $x_B = 0.001$.

3. Consider the QED amplitude $T_{+,\pm}(s,Q^2)$ describing $e(k,+)\mu(p,+) \to e(k,\pm)\mu(p',+)$, where $\pm$ denote helicities, $s=(k+p)^2$ and $Q^2=-(k-k')^2$. Take the high energy limit where $k^+,\ p^- \to \infty$ at fixed Q^2 with $k_\perp = p_\perp = 0$, keeping the masses m_e and m_μ only where they contribute to the leading term. Define

$$T_{+,\pm}(s,Q^2) = -\frac{e^2}{Q^2} g_{\mu\nu} L^\mu_{+,\pm}(k,k')\, L^\nu_{+,+}(p,p') \tag{1}$$

where $L^\mu_{+,\pm}(k,k') = \bar{u}(k',\pm)\gamma^\mu u(k,+)$ and similarly for the muon vertex. Use the LF spinors given by (14) and by (21), (23) in the formula collection for the electron and muon vertices, respectively.

   (a) Show that the leading term of the non-flip vertices is $L^+_{+,+}(k,k') \simeq 2k^+$ and $L^-_{+,+}(p,p') \simeq 2p^-$ and hence find $T_{+,+}(s,Q^2)$ to leading order.

   (b) Show that $L^\mu_{+,-}(k,k') \simeq m_e\sqrt{2}(n^\mu e_+ \cdot q - e^\mu_+ q^+)/k^+$. Find $T_{+,-}(s,Q^2)$ to leading order. Show that the ratio $T_{+,-}(s,Q^2)/T_{+,+}(s,Q^2) \propto 1/s^2$.

   (c) Include the contribution of the anomalous moment of the electron by adding a Pauli coupling $P^\mu_{+,\pm}(k,k') \equiv \bar{u}(k',\pm)i\sigma^{\mu\nu}(k-k')_\nu u(k,+)F_2(Q^2)/2m_e$, where $F_2(Q^2)$ is the Pauli form factor. Show that $T_{+,-}(s,Q^2)/T_{+,+}(s,Q^2)$ is independent of s due the Pauli contribution to $T_{+,-}$. Does this means that the probability of a helicity flip for a physical electron actually does not decrease with the collision energy?

1. $\bar{u}(p+\frac{1}{2}\Delta)\left[2m\,\gamma^{(\mu}p^{\nu)} - 2p^\mu p^\nu - p^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha\right\}u(p-\frac{1}{2}\Delta)$

$$= \bar{u}\left\{2m\,\gamma^{(\mu}p^{\nu)} - 2p^\mu p^\nu + \frac{1}{2}p^{(\mu}\left[\gamma^{\nu)}\not{\Delta} - \not{\Delta}\gamma^{\nu)}\right]\right\}u$$

Use $\Delta = (p+\frac{1}{2}\Delta) - (p-\frac{1}{2}\Delta)$ and $\left[(\not{p}-\frac{1}{2}\not{\Delta}) - m\right]u(p-\frac{1}{2}\Delta) = 0$

$\bar{u}(p+\frac{1}{2}\Delta)\left[(\not{p}+\frac{1}{2}\not{\Delta} - m\right] = 0$

$\Rightarrow$

$$\bar{u}\left\{2m\,\gamma^{(\mu}p^{\nu)} - 2p^\mu p^\nu + \frac{1}{2}p^{(\mu}\gamma^{\nu)}\left[\not{p}+\frac{1}{2}\not{\Delta} - m\right] + \frac{1}{2}p^{(\mu}\left[m - \not{p} + \frac{1}{2}\not{\Delta}\right]\gamma^{\nu)}\right\}u$$

$$= \bar{u}\left\{m\,\gamma^{(\mu}p^{\nu)} - p^\mu p^\nu + \frac{1}{4}p^{(\mu}\left[\gamma^{\nu)}\not{\Delta} - \not{\Delta}\gamma^{\nu)}\right]\right\}u$$

$= \frac{1}{2}$ (original expression) $\Rightarrow$ Original expression $= 0$

2 a)

$$\gamma^* \; (\nu, Q^2) \longrightarrow q\,(z\nu, \vec{p}_\perp) \quad \bar{q}\,((1-z)\nu, -\vec{p}_\perp)$$

$$\Delta E = E_{\gamma^*} - E_q - E_{\bar{q}} = \nu - \sqrt{(z\nu)^2 + p_\perp^2 + m_q^2} - \sqrt{(1-z)^2\nu^2 + p_\perp^2 + m_q^2}$$

$$\simeq -\frac{p_\perp^2 + m_q^2}{2\nu}\left[\frac{1}{z} + \frac{1}{1-z} = \frac{1}{z(1-z)}\right] \approx \frac{1}{\Delta t}$$

$$v_\perp \Delta t \simeq \frac{p_\perp}{z\nu}\,\frac{2\nu\, z(1-z)}{p_\perp^2 + m_q^2} = 2(1-z)\,\frac{p_\perp}{p_\perp^2 + m_q^2}$$

For $p_\perp, m_q \sim \Lambda_{QCD}$: $v_\perp \Delta t \sim 1$ fm

$p_\perp \sim m_q \gg \Lambda_{QCD}$: $v_\perp \Delta t \sim \frac{1}{m_q}$

$p_\perp \gg m_q, \Lambda_{QCD}$: $v_\perp \Delta t \sim \frac{1}{p_\perp}$

$z \to 1$: $v_\perp \Delta t$ for q is small,
but for $\bar{q}$ is as above.

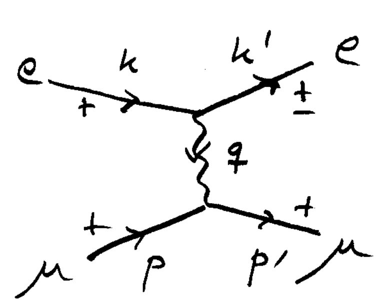
b) γ^* coherence length ~ life-time is Lorentz dilated by its $\gamma = \frac{E}{m}$ factor:

$$L_I \sim \frac{1}{Q}\cdot\frac{\nu}{Q} = \frac{\nu}{Q^2} = \frac{1}{2m_p x_B}$$

$$\simeq \frac{0.2\ \text{fm}}{2\cdot 0.001} = 100\ \text{fm}$$

3 $\quad k = \left(k^+, \frac{m_e^2}{k^+}, \vec{0}_\perp\right)$; $k^+ \to \infty$

$p = \left(\frac{m_\mu^2}{p^-}, p^-, \vec{0}_\perp\right)$; $p^- \to \infty$



$$k'^2 = m_e^2 = (k-q)^2 = m_e^2 - 2k\cdot q - Q^2$$

$$2k\cdot q \simeq k^+ q^- = -Q^2 ; \quad q^- \simeq -\frac{Q^2}{k^+}$$

$$p'^2 = m_\mu^2 = (p+q)^2 = m_\mu^2 + 2p\cdot q - Q^2 \Rightarrow q^+ \simeq \frac{Q^2}{p^-}$$

$\left.\right\}$ $Q^2 \simeq q_\perp^2$, $k'^+ \simeq k^+$, $p'^+ \simeq p^-$

$$iT_{+,\pm}(s,Q^2) = \frac{(-i)^3 e^2}{q^2} g_{\mu\nu} L^\mu_{+,\pm}(k,k') L^\nu_{++}(p,p')$$

a)

$$L^\mu_{+,+}(k,k') = \frac{1}{4k^+} \mathrm{Tr}\left[(\not k' + m_e)\gamma^\mu(\not k + m_e)\not n(1+\gamma_5)\right]$$

Only for $\mu = +$ can Tr be $\propto (k^+)^2$:

$$L^+_{+,+}(k,k') = \frac{1}{4k^+}\mathrm{Tr}\left[\not k' \not n \not k \not n\right] = \frac{2k'^+k^+}{k^+} \simeq 2k^+$$

$$L^-_{++}(p,p') = \frac{1}{4p^-}\mathrm{Tr}\left[(\not p' + m_\mu)\tilde{\not n}(\not p + m_\mu)\tilde{\not n}(1-\gamma_5)\right] \simeq 2p^-$$

$$T_{++}(s,Q^2) \simeq -\frac{e^2}{Q^2}\frac{1}{2} L^+_{++}(k,k') L^-_{++}(p,p') = -\frac{2e^2 \overbrace{k^+p^-}^{s}}{Q^2}$$

b)

$$L^\mu_{+-}(k,k') = -\frac{1}{2\sqrt{2}k^+}\mathrm{Tr}\left[(\not k' + m_e)\gamma^\mu(\not k + m_e)\not n \not e_+\right]$$

$$= -\frac{m_e}{2\sqrt{2}k^+}\mathrm{Tr}\left[\gamma^\mu \not k \not n \not e_+ + \not k' \gamma^\mu \not n \not e_+\right]$$

$$= -\frac{m_e\sqrt{2}}{k^+}\left[e_+^\mu k^+ + n^\mu e_+\cdot k' - e_+^\mu k'^+\right]$$

$$k - k' = q \; ; \; e_+ \cdot k' = -e_+ \cdot q$$

$$L^\mu_{+-}(k,k') = \frac{m_e\sqrt{2}}{k^+}\left[n^\mu\, e_+\cdot q - e_+^\mu\, q^+\right]$$

$$n_\mu L^\mu_{++}(p,p') = \frac{1}{4p^-}\mathrm{Tr}\left[(\not p' + m_\mu)\not n(\not p + m_\mu)\tilde{\not n}(1-\gamma_5)\right]$$

$$= \frac{1}{p^-}\left[2m_\mu^2 + p'^+p^- + p'^-p^+ - 2p\cdot p'\right]$$

$$2m_\mu^2 - 2p\cdot p' = (p-p')^2 = q^2 = -Q^2$$

$$p'^+p^- = \frac{q_\perp^2 + m_\mu^2}{p'^-}\,p^- \simeq Q^2 + m_\mu^2$$

$$p'^-p^+ = p'^-\frac{m_\mu^2}{p^-} \simeq m_\mu^2$$

$$\Rightarrow n_\mu L^\mu_{++}(p,p') \simeq \frac{2m_\mu^2}{p^-}$$

$$e_{+\mu}L^\mu_{++}(p,p') = \frac{1}{4p^-}\mathrm{Tr}\left[(\not p' + m_\mu)\underbrace{\not e_+(\not p + m_\mu)\tilde{\not n}(1-\gamma_5)}_{(-\not p + m_\mu)\underbrace{(-\tilde{\not n})(1+\gamma_5)\not e_+}_{=0 \text{ by } (9)}}\right]$$

$$T_{+-} \simeq -\frac{e^2}{Q^2}\frac{m_e\sqrt{2}}{k^+}\frac{\overbrace{q_\perp}^{Q}}{\sqrt{2}}e^{i\varphi}\frac{2m_\mu^2}{p^-} = -\frac{2e^2}{Q^2}\frac{m_e m_\mu^2}{s}e^{i\varphi}$$

$$\frac{T_{+-}}{T_{++}} \simeq Q\,\frac{m_e m_\mu^2}{s^2}e^{i\varphi}$$ indeed vanishes as $\frac{1}{s^2}$

$\propto m_e$ (helicity flip)

$\propto q_\perp e^{i\varphi}$ ($L_z = 1$, compensates $\Delta S_z = -1$)

3c) Pauli coupling of electron

$$P^\mu_{+-}(k,k') = \bar u(k',-)\, i\,\sigma^{\mu\nu} q_\nu\, u(k,+)\, \frac{F_2(Q^2)}{2m_e}$$

Now $P^+_{+-} \propto k^+$:

$$P^+_{+-}(k,k') = -\frac{1}{2}\,\frac{F_2}{2m_e}\,\frac{-1}{2\sqrt{2}\,k^+}\,\mathrm{Tr}\Big[(\slashed{k}'+\overset{\circ}{m_e})\big(\overbrace{\slashed{\varkappa}\slashed{q}-\slashed{q}\slashed{\varkappa}}^{\simeq\, 2\slashed{\varkappa}\slashed{q}}\big)(\slashed{k}+\overset{\circ}{m_e})\,\slashed{\varkappa}\,\slashed{\epsilon}_+\Big]$$

$$\simeq \frac{F_2}{2\sqrt{2}}\,\frac{k'^+}{m_e k^+}\,\mathrm{Tr}\big[\slashed{q}\,\slashed{k}\,\slashed{\varkappa}\,\slashed{\epsilon}_+\big] \simeq \frac{\sqrt{2}\,F_2}{m_e}\,k^+\,\epsilon_+\cdot q$$

$$= \frac{F_2}{m_e}\,k^+ Q\, e^{i\varphi}$$

This contribution gives

$$T_{+-}(s,Q^2) = -\frac{e^2}{Q^2}\,\frac{F_2}{m_e}\,k^+ Q\, e^{i\varphi}\, 2p^-\cdot\frac{1}{2} = -\frac{e^2 F_2}{Q\,m_e}\, s\, e^{i\varphi}$$

Thus

$$\frac{T_{+-}}{T_{++}} \simeq \frac{Q\,F_2(Q^2)}{2m_e}\, e^{i\varphi}$$

and indeed the electron can flip its helicity in high energy ($s\to\infty$) interactions due to its (small) anomalous magnetic moment, which arises at higher orders:

The flip occurs at one of the $e\to e\gamma$ vertices.