QCD and Hadron Structure Fall 2008 **Exercise 9**

Session of Monday 3 November at 16.00-17.30 in aud A315.

1. (a) At high energy and low momentum transfer, for coherent scattering on the entire nuclear target A, the measured cross section of elastic J/ψ photoproduction on nuclei, $\sigma(\gamma + A \to J/\psi + A) \propto A^{1.40}$ (*cf.* Hoyer-2 p. 18). Explain why one would expect the A-dependence to be roughtly $\sigma \propto A^{4/3}$, assuming that the amplitude for scattering on a single nucleon $T(\gamma N \to J/\psi + N) = c\, s \exp(-bq_\perp^2)$, where q is the momentum transfer, $s = E_{CM}^2$ and b, c are constants.

   (b) Estimate the hadron formation length (in fm) for a quark in DIS carrying $\nu = 50$ GeV. (*Cf.* the data on Hoyer-2 p. 20.)

2. Elastic ($ab \to ab$) hadron scattering amplitudes at small momentum transfers t are found to be approximately proportional to $s = (p_a + p_b)^2$, which through the optical theorem corresponds to a roughly constant total cross section $\sigma_{tot}(ab)$. In the Regge model this is ascribed to "Pomeron exchange",

$$T(s,t) = \beta(t) \left(\frac{s}{s_0}\right)^{\alpha_{\mathbb{P}}(t)} \exp(i\phi) \tag{1}$$

   where $\beta(t)$ is a real function, $s_0 \simeq 1$ GeV, $\alpha_{\mathbb{P}}(0) \simeq 1$ and ϕ is a phase.

   (a) On general grounds we know that the amplitude is a real analytic function of s, *i.e.*, that $A(s^*, t) = A^*(s,t)$ for $t < 0$. The physical region for $s > (m_a + m_b)^2$ is above the unitarity cut ($s + i\varepsilon$) and analogously at $u + i\varepsilon$ in the crossed channel $\bar{a}b \to \bar{a}b$ where $u > (m_a + m_b)^2$ (s and u are linearly dependent at fixed t due to $s + t + u = 2m_a^2 + 2m_b^2$). Show that $\phi = -\pi\alpha_{\mathbb{P}}(t)/2$ when $a = \bar{a}$, *i.e.*, when the amplitude is $s \leftrightarrow u$ symmetric.

   (b) Explain the statement on Hoyer-2 p. 25 that with $W^2 = s$,

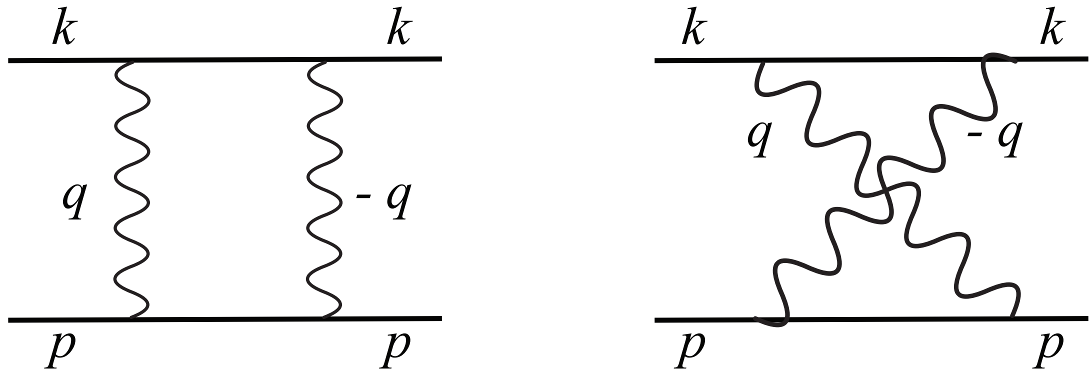
$$\frac{d\sigma_{diff}^{\gamma^* p}/dM_X}{\sigma_{tot}^{\gamma^* p}} \propto \frac{(W^2)^{2\bar{\alpha}_{\mathbb{P}}-2}}{(W^2)^{\alpha_{\mathbb{P}}(0)-1}}$$

3. According to your result 2(a) above the phase $\exp(i\phi) = -i$ for a crossing-symmetric amplitude $T \propto s$. This can be verified for the two-photon exchange contribution to the $e(k)\mu(p) \to e(k)\mu(p)$ amplitude (fig. 1 on next page) in the limit k^+, $p^- \to \infty$. For simplicity we may consider the forward direction and neglect the e and μ masses. While $e^+ \neq e^-$ this contribution is still $s \leftrightarrow u$ symmetric since it is proportional to the square of the electron charge.

   (a) Write the Feynman gauge expressions for the two diagrams in fig. 1.

   (b) Show that the real parts of both diagrams are $\propto s$ and cancel to leading order in s. You may assume that the leading contributions in the $s \to \infty$ limit come from the region of fixed (independent of s) loop momentum q.

   (c) Identify the imaginary part using the Cutkosky rule, and show that it is also $\propto s$. Thus the general result in 2(a) holds also for this QED amplitude.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