

Session of Monday 3 November at 16.00-17.30 in aud A315.

1. (a) At high energy and low momentum transfer, for coherent scattering on the entire nuclear target A , the measured cross section of elastic J/ψ photoproduction on nuclei, $\sigma(\gamma + A \rightarrow J/\psi + A) \propto A^{1.40}$ (cf. Hoyer-2 p. 18). Explain why one would expect the A -dependence to be roughly $\sigma \propto A^{4/3}$, assuming that the amplitude for scattering on a single nucleon $T(\gamma N \rightarrow J/\psi + N) = c s \exp(-bq_{\perp}^2)$, where q is the momentum transfer, $s = E_{CM}^2$ and b, c are constants.

(b) Estimate the hadron formation length (in fm) for a quark in DIS carrying $\nu = 50$ GeV. (Cf. the data on Hoyer-2 p. 20.)
2. Elastic ($ab \rightarrow ab$) hadron scattering amplitudes at small momentum transfers t are found to be approximately proportional to $s = (p_a + p_b)^2$, which through the optical theorem corresponds to a roughly constant total cross section $\sigma_{tot}(ab)$. In the Regge model this is ascribed to "Pomeron exchange",

$$T(s, t) = \beta(t) \left(\frac{s}{s_0} \right)^{\alpha_{\mathbb{P}}(t)} \exp(i\phi) \quad (1)$$

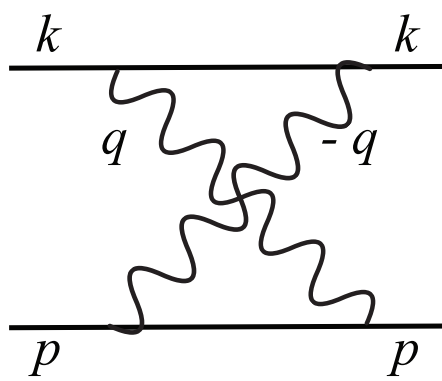
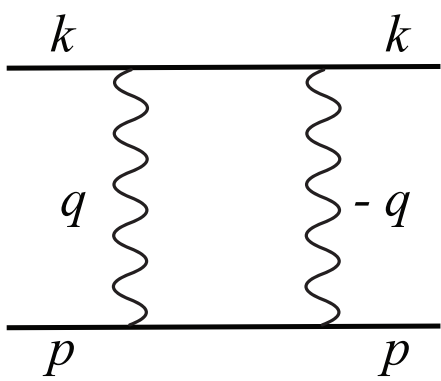
where $\beta(t)$ is a real function, $s_0 \simeq 1$ GeV, $\alpha_{\mathbb{P}}(0) \simeq 1$ and ϕ is a phase.

- (a) On general grounds we know that the amplitude is a real analytic function of s , *i.e.*, that $A(s^*, t) = A^*(s, t)$ for $t < 0$. The physical region for $s > (m_a + m_b)^2$ is above the unitarity cut ($s + i\varepsilon$) and analogously at $u + i\varepsilon$ in the crossed channel $\bar{a}b \rightarrow \bar{a}b$ where $u > (m_a + m_b)^2$ (s and u are linearly dependent at fixed t due to $s + t + u = 2m_a^2 + 2m_b^2$). Show that $\phi = -\pi\alpha_{\mathbb{P}}(t)/2$ when $a = \bar{a}$, *i.e.*, when the amplitude is $s \leftrightarrow u$ symmetric.

(b) Explain the statement on Hoyer-2 p. 25 that with $W^2 = s$,

$$\frac{d\sigma_{diff}^{\gamma^*p}/dM_X}{\sigma_{tot}^{\gamma^*p}} \propto \frac{(W^2)^{2\alpha_{\mathbb{P}}-2}}{(W^2)^{\alpha_{\mathbb{P}}(0)-1}}$$

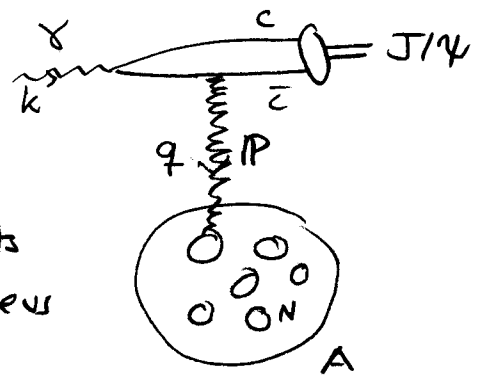
3. According to your result 2(a) above the phase $\exp(i\phi) = -i$ for a crossing-symmetric amplitude $T \propto s$. This can be verified for the two-photon exchange contribution to the $e(k)\mu(p) \rightarrow e(k)\mu(p)$ amplitude (fig. 1 on next page) in the limit $k^+, p^- \rightarrow \infty$. For simplicity we may consider the forward direction and neglect the e and μ masses. While $e^+ \neq e^-$ this contribution is still $s \leftrightarrow u$ symmetric since it is proportional to the square of the electron charge.
 - (a) Write the Feynman gauge expressions for the two diagrams in fig. 1.
 - (b) Show that the real parts of both diagrams are $\propto s$ and cancel to leading order in s . You may assume that the leading contributions in the $s \rightarrow \infty$ limit come from the region of fixed (independent of s) loop momentum q .
 - (c) Identify the imaginary part using the Cutkosky rule, and show that it is also $\propto s$. Thus the general result in 2(a) holds also for this QED amplitude.



1. a) The transverse radius of the $c\bar{c}$ fluctuation

$$r_{\perp}(c\bar{c}) \approx \frac{1}{m_c} \ll \Lambda_{\text{QCD}}$$

Being a color singlet, it interacts weakly (once) in the target nucleus



* At high energy ($k^+ \rightarrow \infty$) the longitudinal momentum transfer required for $\gamma \rightarrow J/\psi$ is small:

$$M_{J/\psi}^2 = (k - q)^2 = -k^+ q^- + q^2 \Rightarrow q^- = \frac{q^2 - M_{J/\psi}^2}{k^+} \rightarrow 0$$

* Coherent scattering over the nucleus with $R_A \approx A^{1/3}$ fm requires $q_{\perp} \lesssim \frac{1}{R_A}$

* If $T(\gamma N \rightarrow J/\psi N) \approx c s e^{-b q_{\perp}^2}$ (c, b constants)

$\Rightarrow T(\gamma A \rightarrow J/\psi A) \approx A c s e^{-b q_{\perp}^2}$ ($q_{\perp} \lesssim 1/R_A$)

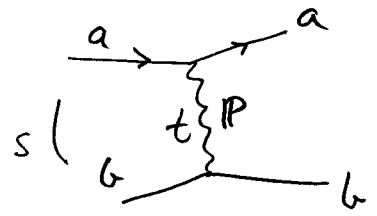
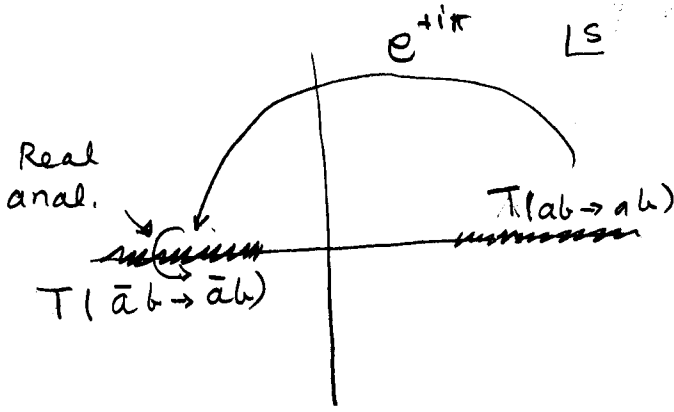
$$\frac{\sigma(\gamma A \rightarrow J/\psi A)}{\sigma(\gamma N \rightarrow J/\psi N)} \approx A^2 \frac{\int_0^{1/R_A^2} dq_{\perp}^2 e^{-2b q_{\perp}^2}}{\int_0^{\infty} dq_{\perp}^2 e^{-2b q_{\perp}^2}} \approx A^2 \frac{2b}{R_A^2} \propto A^{1/3}$$

b) Formation length at rest: $L_0 \approx \frac{1}{\Lambda_{\text{QCD}}}$

— " — $v : L \approx \gamma \cdot L_0 \approx \frac{v}{\Lambda_{\text{QCD}}} \cdot \frac{1}{\Lambda_{\text{QCD}}}$

$$\Lambda_{\text{QCD}} \approx 200 \text{ MeV} \approx \frac{1}{1 \text{ fm}} \quad \left| \quad \begin{aligned} &\approx 50 \text{ GeV} (1 \text{ fm})^2 \frac{1}{0.2 \text{ GeV fm}} \\ &\approx 250 \text{ fm} \end{aligned} \right.$$

2 a)



$$u = 2m_a^2 + 2m_b^2 - t - s$$

$$u + i\epsilon \leftrightarrow s - i\epsilon$$

$$T(ab \to ab) = T(\bar{a}b \to \bar{a}b)$$

$$= \beta(t) \left(\frac{s}{s_0}\right)^{\alpha_P(t)} e^{i\varphi} \quad (s \to +\infty)$$

$$\rightarrow \beta(t) \left(\frac{|s|}{s_0}\right)^{\alpha_P(t)} e^{+i\pi\alpha_P} e^{i\varphi} \quad \left\{ \begin{array}{l} (s \rightarrow |s| e^{i\pi}) \rightarrow -\infty \\ u \rightarrow \infty - i\epsilon \end{array} \right.$$

$$\text{(Real analyticity)} \rightarrow \beta(t) \left(\frac{u}{s_0}\right)^{\alpha_P(t)} e^{-i\pi\alpha_P} e^{-i\varphi} \quad u \rightarrow \infty + i\epsilon$$

$$= T(\bar{a}b \to \bar{a}b)$$

$$\Rightarrow e^{i\varphi} = e^{-i\pi\alpha_P(t)} e^{-i\varphi} \quad ; \quad \varphi = -\frac{\pi}{2} \alpha_P(t)$$

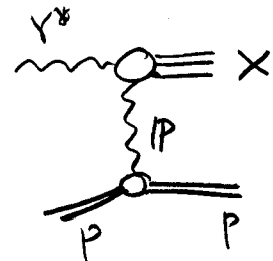
$$2b) \quad \sigma_{tot}(\gamma^* p) = \frac{1}{2s} \text{Im} T(\gamma^* p \rightarrow \gamma^* p) \Big|_{t=0} \propto s^{\alpha_P(0)-1}$$

$$\frac{d\sigma_{diff}}{dt}(\gamma^* p \rightarrow X p) \propto \frac{1}{s^2} |T|^2$$

$$\propto s^{2\alpha_P(t)-2}$$

$$\int dt : \alpha_P(t) \rightarrow \bar{\alpha}_P$$

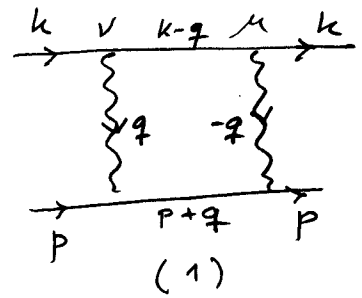
$$\Rightarrow \frac{d\sigma_{diff}^{\gamma^* p} / dM_X}{\sigma_{tot}^{\gamma^* p}} \propto \frac{s^{2\bar{\alpha}_P-2}}{s^{\alpha_P(0)-1}}$$



$$\alpha_P(0) \approx 1.08$$

$$s = W^2$$

3a) $k = (k^+, 0^-, \vec{0}_\perp)$
 $p = (0^+, p^-, \vec{0}_\perp)$; $s = k^+ p^-$



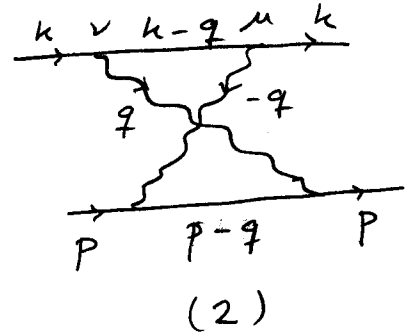
$$i T_1 = (-ie)^4 i^2 (-i)^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 + i\epsilon)^2}$$

$$* \bar{u}(k) \gamma^\mu \frac{\not{k} - \not{q}}{(k-q)^2 + i\epsilon} \gamma^\nu u(k) \bar{u}(p) \gamma_\mu \frac{\not{p} + \not{q}}{(p+q)^2 + i\epsilon} \gamma_\nu u(p)$$

$$T_2 = -ie^4 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 + i\epsilon)^2}$$

$$* \bar{u}(k) \gamma^\mu \frac{\not{k} - \not{q}}{(k-q)^2 + i\epsilon} \gamma^\nu u(k)$$

$$* \bar{u}(p) \gamma_\nu \frac{\not{p} - \not{q}}{(p-q)^2 + i\epsilon} \gamma_\mu u(p)$$



$$d^4 q = \frac{1}{2} dq^+ dq^- d^2 \vec{q}_\perp$$

3b) $(k-q)^2 = -k^+ q^- + q^+ q^- - q_\perp^2 \simeq -k^+ q^-$ (fixed q)

$(p+q)^2 = p^- q^+ + q^+ q^- - q_\perp^2 \simeq p^- q^+$ (fixed q)

Leading behavior for $\gamma^\mu = \gamma^\nu = \gamma^+ = \not{x}$:

$$\frac{1}{4k^+} \text{Tr} [\not{k} \not{x} (\not{k} - \not{q}) \not{x} \not{k} \not{x} (1 \pm \gamma_5^0)] = \frac{(2k^+)^2}{4k^+} \text{Tr} [(\not{k} - \not{q}) \not{x}]$$

$$\simeq 4(k^+)^2$$

$$\frac{1}{4p^-} \text{Tr} [\not{p} \not{x} (\not{p} + \not{q}) \not{x} \not{p} \not{x} (1 \mp \gamma_5^0)] \simeq 4(p^-)^2$$

$(k-q)^2 (p+q)^2 \simeq -s q^- q^+$ (fixed q)

$$T_1 \approx -ie^4 \frac{(2k^+)^2 (2p^-)^2}{-s} \frac{1}{2} \frac{1}{2} \int \frac{dq^+ dq^- d^2 \vec{q}_\perp}{(2\pi)^4} \frac{1}{q^+ q^-} \frac{1}{(q^2 + i\epsilon)^2}$$

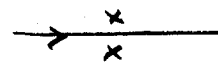
$$= 4ie^4 s \int \frac{dq^+ dq^- d^2 \vec{q}_\perp}{(2\pi)^4} \frac{1}{q^+ q^-} \frac{1}{(q^2 + i\epsilon)^2}$$

In T_2 : $(p-q)^2 = -p^- q^+ + q^+ q^- - \vec{q}_\perp^2 \approx -p^- q^+$ (fixed q)

All other factors the same as in T_1

$$\Rightarrow T_2 \approx -T_1 \quad \text{Real parts cancel (to leading order in } s)$$

3c) Imaginary part arises from pinch of integration contour: Poles on opposite sides, cannot distort contour



Also: $(k-q)^- > 0 \Rightarrow q^- < 0$

$(p+q)^+ > 0 \Rightarrow q^+ > 0$ (in T_1)

T_1 :

$$(k-q)^2 + i\epsilon = -k^+ q^- + q^+ q^- - \vec{q}_\perp^2 + i\epsilon = 0 \Rightarrow \text{Im } q^- > 0$$

$$(p+q)^2 + i\epsilon = p^- q^+ + q^+ q^- - \vec{q}_\perp^2 + i\epsilon = 0 \Rightarrow \text{Im } q^- < 0$$

\Rightarrow Pinch in $\int dq^-$ when both denominators vanish

$$\text{Im } T_1 \neq 0$$

T_2 : $(p-q)^+ > 0 \Rightarrow q^+ < 0$

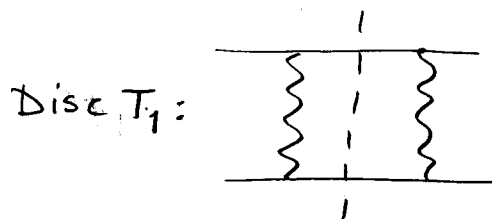
$$\text{Im } T_2 = 0$$

$$(p-q)^2 = -p^- q^+ + q^+ q^- - \vec{q}_\perp^2 + i\epsilon = 0 \Rightarrow \text{Im } q^- > 0 \quad \text{No pinch!}$$

3c cont.) Cutkosky rule: In T_2 ,

cut would cross also photon lines, but these cannot be simultaneously on-shell $\Rightarrow \text{Im } T_2 = 0$

$$\frac{1}{(k-q)^2 + i\epsilon} \rightarrow -2\pi i \delta[(k-q)^2]$$



$$\frac{1}{(p+q)^2 + i\epsilon} \rightarrow -2\pi i \delta[(p+q)^2]$$

$$(-2\pi i)^2 \frac{1}{2} \int \frac{dq^+ dq^-}{(2\pi)^2} \delta(+k^+ q^- + q^2) \delta(p^- q^+ + q^2) = \frac{-1}{2k^+ p^-}$$

$$\text{Disc } T_1 = 2i \text{Im } T_1 = -ie^4 \frac{1}{2} (2k^+)^2 (2p^-)^2 \frac{-1}{2k^+ p^-} \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \frac{1}{q_\perp^4}$$

$$\text{Im } T_1 = \text{Im } T_1 = 2e^4 s \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \frac{1}{q_\perp^4}$$