1. Show that if two relativistic particles are described by $p_1 = (E_1, \vec{p}_1)$ and $p_2 = (E_2, \vec{p}_2)$ in a Lorentz frame in which they move collinearly with the relative velocity $\vec{v}_{rel}$, then
$$E_1 E_2 v_{rel} = [(p_1 p_2)^2 - m_1^2 m_2^2]^{1/2}.$$
Hint: Starting from the relativistic relation between momentum and velocity, $\vec{p} = m\vec{v}/\sqrt{1-|\vec{v}|^2}$, show that $\vec{v} = \vec{p}/E$.

2. Consider the following QED processes, in the second order of perturbation:
$$\begin{aligned} \text{i)} \quad & e^+e^- \to e^+e^-; \\ \text{ii)} \quad & e^+e^+ \to e^+e^+; \\ \text{iii)} \quad & e^+e^- \to \gamma\gamma. \end{aligned}$$
Draw the corresponding Feynman diagrams and write the Feynman amplitudes using the Feynman rules. For the processes which comprise two diagrams, emphasize the relative sign between the respective Feynman diagrams.

3. Draw the Feynman diagrams and calculate the unpolarized differential cross-section for the Bhabha scattering in QED,
$$e^+ + e^- \to e^+ + e^-,$$
in the lowest order in perturbation theory, in the high-energy limit ($E \gg m$) and in the center-of-mass frame. Show that it is equal to
$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\alpha^2}{8E^2}\left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} + \frac{1+\cos^2\theta}{2} - \frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)}\right],$$
where α is the fine structure constant, θ is the scattering angle and E is the energy of either fermion in the center-of-mass frame.

   Hint: Since you are interested in the high-energy limit, you can neglect already in the computation of $|\overline{\mathcal{M}}|^2$ the terms proportional to m^2 and m^4, i.e. there is no need to calculate the corresponding traces.

Please return the solved problems into the box on the second floor of the Physicum building by Wednesday 7th December by 18:00.