Useful formulas:

$$\int_0^\infty \mathrm{d}x\, x^{\alpha-1} e^{-x} = 2\int_0^\infty \mathrm{d}x\, x^{2\alpha-1} e^{-x^2} = \Gamma(\alpha)$$

$\Gamma(\alpha+1) = \alpha\Gamma(\alpha); \quad \Gamma(1) = 1, \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$

$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]; \ \cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$
$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$

Spherical coordinates : $x = r\sin\theta\cos\varphi; \quad y = r\sin\theta\sin\varphi; \quad z = r\cos\theta$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

Flux density: $\mathbf{\Gamma} = \frac{-i\hbar}{2m}(\psi^*\boldsymbol{\nabla}\psi - \psi\boldsymbol{\nabla}\psi^*)$

Potential well: $V(|x|<L) = 0; \quad V(|x|>L) = \infty$: $E_n = (\hbar n\pi)^2/8mL^2$
$u_n^{(+)}(x) = \cos(\pi n x/2L)/\sqrt{L}$ (odd n); $u_n^{(-)}(x) = \sin(\pi n x/2L)/\sqrt{L}$ (even n)

Harmonic oscillator:
$V = \frac{1}{2}m\omega^2x^2, \quad \psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\exp(-\frac{m\omega}{2\hbar}x^2), \quad a = \sqrt{\frac{m\omega}{2\hbar}}\left(x + \frac{ip}{m\omega}\right)$
$E_n = \hbar\omega(n+\frac{1}{2}), \quad a|n\rangle = \sqrt{n}|n-1\rangle, \quad a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$

Hydrogen atom: $u_{n\ell m}(\mathbf{r}) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi)$

$R_{10}(r) = \frac{2Z^{3/2}}{a_0^{3/2}}e^{-Zr/a_0}, \quad R_{20}(r) = \frac{Z^{3/2}}{2\sqrt{2}a_0^{3/2}}(2 - \frac{Zr}{a_0})e^{-Zr/2a_0},$

$R_{21}(r) = \frac{Z^{5/2}}{2\sqrt{6}a_0^{5/2}}re^{-Zr/2a_0}, \quad E_n \overset{\text{gauss}}{=} -\left(\frac{Ze^2}{\hbar}\right)^2\frac{m_r}{2}\frac{1}{n^2} \overset{\text{SI}}{=} -\left(\frac{Ze^2}{4\pi\epsilon_0\hbar}\right)^2\frac{m_r}{2}\frac{1}{n^2}$

$Y_{00}(\theta,\varphi) = \sqrt{1/4\pi}, \quad Y_{10}(\theta,\varphi) = \sqrt{3/4\pi}\,\cos\theta, \quad Y_{1\pm1}(\theta,\varphi) = \mp\sqrt{3/8\pi}\,\sin\theta\, e^{\pm i\varphi}$
$Y_{20}(\theta,\varphi) = \sqrt{5/16\pi}\,(3\cos^2\theta - 1), \quad Y_{2,\pm1}(\theta,\varphi) = \mp\sqrt{15/8\pi}\,\sin\theta\cos\theta\, e^{\pm i\varphi},$
$Y_{2\pm2}(\theta,\varphi) = \sqrt{15/32\pi}\,\sin^2\theta\, e^{\pm 2i\varphi}$

Energy and wave function in perturbation theory: $H = H_0 + gV$

$$E_n = E_n^{(0)} + g\langle\psi_n^{(0)}|V|\psi_n^{(0)}\rangle + g^2\sum_{\ell\neq n}\frac{|\langle\psi_\ell^{(0)}|V|\psi_n^{(0)}\rangle|^2}{E_n^{(0)} - E_\ell^{(0)}} + \mathcal{O}\left(g^3\right)$$

$$|\psi_n\rangle = |\psi_n^{(0)}\rangle + g\sum_{\ell\neq n}\frac{\langle\psi_\ell^{(0)}|V|\psi_n^{(0)}\rangle}{E_n^{(0)} - E_\ell^{(0)}}|\psi_\ell^{(0)}\rangle + \mathcal{O}\left(g^2\right)$$

Born approximation for the scattering amplitude in a potential $V(\mathbf{r})$:

$$f_{\mathbf{k}}(\Omega_{\mathbf{k}'}) = -\frac{m}{2\pi\hbar^2}\int d^3\mathbf{r}\, e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\, V(\mathbf{r})$$

Pauli matrices: $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$a_0 \overset{\text{gauss}}{=} \frac{\hbar^2}{me^2} \overset{\text{SI}}{=} \frac{4\pi\epsilon_0\hbar^2}{me^2} \approx 0.53\,\text{Å}\ ; \qquad \alpha \overset{\text{gauss}}{=} \frac{e^2}{\hbar c} \overset{\text{SI}}{=} \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}\,,$$

$\hbar = 1.05457\cdot10^{-34}\,\text{Js} = 6.58212\cdot10^{-16}\,\text{eVs}, \quad \hbar c = 197\ \text{MeV fm}, \quad mc^2 = 0.511\,\text{MeV}$

$m = 9.11\cdot10^{-31}\text{kg}, \quad e = 1.6\cdot10^{-19}\ \text{C}, \quad \epsilon_0 = 8.85\cdot10^{-12}\,\text{C}^2/\text{N}/\text{m}^2.$