KVANTTIMEKANIIKKA I Autumn 2011 Exercise 2

Due at 8.15 on Thursday 22 September 2011

Note: I horribly overestimated how far I could go on Friday's lecture, and the two last probles (3. and 4.) in the hand-out set are beyond what we have gone through. Therefore this week you only have to solve the following two problems:

1. Consider a potential with an imaginary part, *i.e.* $V(x) \to V(x) = V_0(x) - i\Gamma$, where Γ is a positive constant. Show that in this case the total probability P is *not* conserved, as was shown on the lecture, but instead we get

$$\frac{dP}{dt} = -\frac{2\Gamma}{\hbar} P.$$

   Solve $P(t)$ from here and interpret.

2. The wave function of a one-dimensional system is of the form $\psi(x) = Ne^{-ax^2/2}$, with $a > 0$. Calculate:

   (a) normalization coefficient N

   (b) $\langle x^n \rangle$, $n \geq 1$. Thus, what is $\Delta x = \left(\langle x^2 \rangle - \langle x \rangle^2\right)^{1/2}$?

   (c) $\langle p^n \rangle$, $n \geq 1$. (Hint: calculate with wave functions $\varphi(p)$ in the momentum representation or use Hermite polynomials.)

   (d) $\Delta p = \left(\langle p^2 \rangle - \langle p \rangle^2\right)^{1/2}$. Thus, what is $\Delta x \Delta p$?