KVANTTIMEKANIIKKA I Autumn 2011 Exercise 3

Due at 8.15 on Thursday 29 September 2011

Only 4 problems this time, as they require a bit more work. Problem 2 is worth 5 points.

1. A particle sits in an infinitely deep one-dimensional potential well with walls at points $|x| = L$. At time $t = 0$ it is in the state

$$\psi(x,0) = \frac{1}{\sqrt{2}}u_1(x) + \frac{1}{\sqrt{2}}u_2(x),$$

   where $u_1(x)$ and $u_2(x)$ are the normalized ground and first excited state wave functions (these are assumed known). Determine $\langle x \rangle$ and $\langle E \rangle$ at arbitrary time $t > 0$ and also the probability that $x > 0$.

2. (5p) Particle moving freely ($\hat{H} = \hat{p}^2/2m$) is described by the wave packet

$$\psi(x) = \sqrt{\frac{1}{a\sqrt{2\pi}}} \exp\left[-\frac{(x-x_0)^2}{4a^2} + \frac{i}{\hbar}p_0(x-x_0)\right].$$

   (Note that this is concentrated around x_0 and proceeds with an average momentum p_0 as in problem set 1.) Calculate its Fourier transform $\varphi(p)$ and consider that these wave functions represent the system at $t = 0$. Use now the Fourier expansion backwards to calculate the time dependent wave function

$$\Psi(x,t) = \int \frac{\mathrm{d}p}{\sqrt{2\pi\hbar}}\, \varphi(p) \exp\left[-\frac{ip^2}{2m\hbar}t + \frac{i}{\hbar}px\right].$$

   What happens to the probability density as time increases? (Show that it spreads.)

3. A particle with mass m moves in a one-dimensional potential

$$V(x) = \begin{cases} \infty, & x < 0 \\ 0, & 0 < x < L \\ V_0, & x > L \end{cases}$$

   What conditions should a wave function satisfy at the points $x = 0$, L and ∞ in order to represent a bound state? Show that there is a minimum value of V_0 for the existence of a bound state. What is this value?

4. Consider a simple model for a diatomic molecule, with the electron moving in the one-dimensional attractive potential consisting of two δ functions

$$V(x) = -\frac{\hbar^2}{m}\Omega\left[\delta(x-a) + \delta(x+a)\right].$$

   Does the system have any bound states? How do they look like?