KVANTTIMEKANIIKKA I Autumn 2011 Exercise 4

Due at 8.15 on Thursday 6 October 2011

1. A beam of particles (plane wave) meets a potential barrier ($V_0 > 0$)

$$V(x) = \begin{cases} 0, & x < 0 \\ V_0, & x > 0. \end{cases}$$

Compute the transmission and reflection coefficients T and R for different energies and verify that the probability is conserved.

2. Determine the reflection and transmission coefficients in one-dimensional scattering from the δ-function potential

$$V(x) = \frac{\hbar^2}{2m}\Omega\delta(x),$$

where $\Omega > 0$ is a positive constant of dimension $[\Omega] = \text{length}^{-1}$.

3. The wave function of a particle is $\psi(\mathbf{r}) = (2x + z)f(r)$, where $\int_0^\infty \mathrm{d}r\, r^4 |f(r)|^2 = N^2$. What are possible values when measuring $\mathbf{L}^2$ and L_z, and what are their probabilities?

4. A particle is in the ground state of the one-dimensional potential

$$V(x) = \begin{cases} \frac{1}{2}m\omega^2 x^2, & x > 0 \\ \infty, & x < 0. \end{cases}$$

   (a) What is the ground state energy?

   (b) What are the expectation value $\langle x \rangle$ and the most probable value of x?

   (c) If the wall at the origin suddenly vanishes and the potential is now $\frac{1}{2}m\omega^2 x^2$ for all x, what are the probabilities for finding the particle in the new ground state and the first excited state?

   (d) How is this problem relevant for studying a three-dimensional isotropic harmonic oscillator?

5. Consider a particle in a spherically symmetric box,

$$V(x) = \begin{cases} 0, & r < a \\ \infty, & r > a. \end{cases}$$

Solve the bound state energies and wave functions exactly as far as possible. That is, derive an equation, whose solution is the energy, and give the corresponding wave function. If the particle in question is a proton, and $a = 6$ fm, give the numerical value of energy for the ground state and two lowest excited states. (The zeros of Bessel functions can be found e.g. in Abramowitz & Stegun, see link on the course home page.)