

Due at 8.15 on Thursday 3 November 2011

1. Consider a particle in a box on the interval $[-L, L]$. In order to have a more realistic potential, round the bottom of the box using a quadratic perturbation $V(x) = gx^2$. Compute the change in the ground state energy and wave function to first order in perturbation.
2. Hamiltonian for a relativistic harmonic oscillator could be

$$H = \sqrt{p^2c^2 + m^2c^4} + \frac{1}{2}m\omega^2x^2.$$

Using this, compute the lowest order relativistic correction to harmonic oscillator energy levels.

3. Let the moment of inertia of a molecule rotating around z -axis (*i.e.* rotation in xy -plane) be I and its electric dipole moment d . The unperturbed equation for rotational energy eigenstates is then

$$-\frac{\hbar^2}{2I} \frac{d^2\psi(\varphi)}{d\varphi^2} = E\psi(\varphi).$$

Compute using second order perturbation theory the change in the molecule's energy, when it is perturbed by a weak homogeneous electric field in the direction of x -axis, *i.e.* the perturbation potential is $V(\varphi) = -d\mathcal{E} \cos \varphi$.

4. Estimate the effect of a constant electric field on the hydrogen ground state energy using second order perturbation theory (quadratic Stark effect). In the sum over "intermediate" excited states you can restrict to the one state which you think is most important. Try to qualitatively argue why other states should have smaller effect.

Extra: if you find yourself craving for more things to compute, you can try the following (without extra points, however):

- (easy) Derive the equation given in problem 3, starting from time independent Schrödinger equation in polar coordinates, for two particles of same mass at fixed relative distance (so that they only rotate).
- (hard) In problem 4, compute the whole sum over intermediate states. This is not at all trivial, options include going to parabolic coordinates or solving $|\psi^{(1)}\rangle$ directly, not as a series of $\psi_{n\ell m}$. For both methods, see e.g. Schiff.