KVANTTIMEKANIIKKA I Autumn 2011 Exercise 8

Due at 8.15 on Thursday 10 November 2011

1. Estimate using the variational method the energies of two lowest states for a particle of mass m in one-dimensional potential $\lambda|x|$, $\lambda > 0$.

2. Same as the previous problem, but this time estimate energies using perturbation theory: take harmonic oscillator as a starting point and treat the difference in potentials as perturbation,

$$H = H_0 + H_1 = \left(\frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2\right) + \left(\lambda|x| - \frac{1}{2}m\omega^2 x^2\right)$$

   Compute the first order correction to ground state energy E_0. (If you are interested, you can also compute the correction to the energy of the first excited state, to be able to compare with the previous problem, but it is not required.)

   There is some freedom in choosing the parameter ω here. A popular choice in perturbation theory is *the fastest apparent convergence*: choose ω so that the highest computed correction (in this case the first order) vanishes, $E_0^{(1)} = 0$. If you fix ω this way, what is the ground state energy?

3. (5p) The linear potential in problems 1 & 2 is exactly solvable in terms of Airy functions $\mathrm{Ai}(x)$. Literature/Alpha/Wikipedia article on those has all you need, solve the energy states (in terms of zeros of $\mathrm{Ai}, \mathrm{Ai}'$) by hand or using symbolic program of your choice (Maple, Mathematica, Maxima,…) (it may be useful to switch into dimensionless variables first). Note how the symmetry of the potential is again realized in the parity of states, all states are odd or even in x. Compare the exact ground state energy with that from approximation methods (you can get the numeric values of zeros of Ai and Ai′ from e.g. Abramowitz & Stegun).

4. Show that the parity operator P is Hermitean. Is it also unitary? Further show that $(1 + P)/2$ is a projection operator. Is $(1 - P)/2$ also a projection operator? Explain what do they "project" and where? (A projection operator is a Hermitean operator which is *idempotent*, *i.e.* $P^2 = P$.)