Results from 3d Electroweak phase transition simulations

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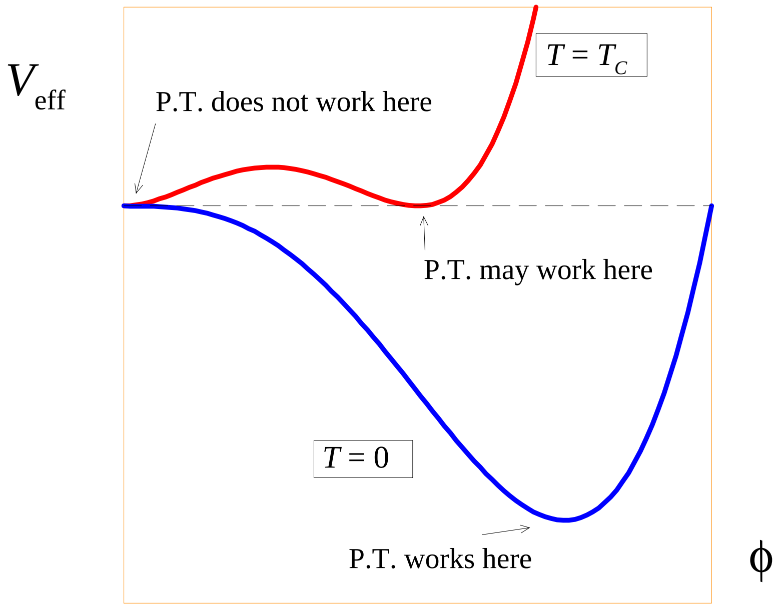
- Standard model has a symmetry restoring/breaking phase transition at $T \sim 140 - 200$ GeV (assuming Higgs mass at $T = 0$ is small enough).
- Baryon number violation in the early Universe? B generation?
- Physical quantities: T_c, latent heat L, surface tension σ, Higgs expectation value $v(T)$, W and Higgs masses $m_W(T)$ and $m_H(T)$ before and after the transition.
- Here we report studies (in 3d formalism):
  - $m_H = 35$, 60 and 70 GeV
  - Several lattice sizes: $V \longrightarrow \infty$
  - lattice spacings $aT = \frac{9}{5}, \frac{9}{8}, \frac{9}{12}, \frac{9}{20}$: continuum limit $a \longrightarrow 0$
- Experimental limit: $m_H \gtrsim 62$ GeV

Other studies:

- $m_H = 35, 80$ GeV: K. Kajantie, K. Rummukainen, M. Shaposhnikov, Nucl.Phys.B 407 (1993) 356
- $m_H = 80$ GeV: K. Farakos, K.K, K.R, M.S, Nucl. Phys. B 425 (1994) 67

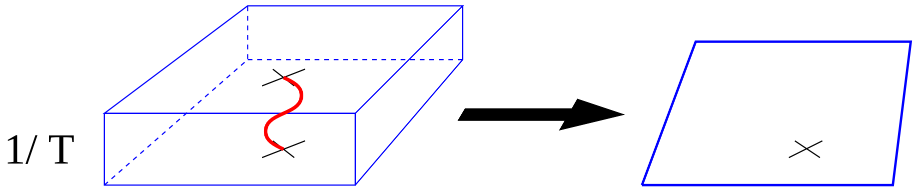
- B. Bunk, E.-M. Ilgenfritz, J. Kripfganz, A. Schiller, Phys. Lett. B284 (1992) 371; Nucl. Phys. B403 (1993) 453
- $m_H = 16$–49 GeV: F. Csikor, Z. Fodor, J. Hein, K. Jansen, A. Jaster, I. Montvay, Phys. Lett. B334 (1994) 405;
  Z. Fodor, J. Hein, K. Jansen, A. Jaster, I. Montvay, Nucl. Phys. B439 (1995) 147
- $m_H = 35$ GeV, 3D, only ϕ: F. Karsch, T. Neuhaus, A. Patkós, Nucl. Phys. B441 (1995) 629
- $m_H = 35$ GeV: F. Csikor, Z. Fodor, J. Hein, J. Heitger, CERN-TH-95-170
- $m_H = 35$ GeV; 3D: E.-M. Ilgenfritz, J. Kripfganz, H. Perlt, A. Schiller, HU Berlin-IEP-95/7

At $T = 0$ perturbation theory works extremely well. Why lattice calculations? $m_W(t) = \frac{1}{2} g v(T)$ acts as an IR regulator $\longrightarrow$ IR divergences in the symmetric phase.



P.T. (2–loop) predicts a first order phase transition for small Higgs masses $m_H \lesssim$70–90 GeV. For large m_H the transition vanishes.

Dimensional Reduction:
Integrate out all massive modes (not constant in τ) from the 4D theory – both bosonic and fermionic.



$$S_4(\varphi_4) \longmapsto S_3(\varphi_3)$$

- Scales:

4D:	$1/T \quad \gg a \gg 1/(m_H(T)L_x)$
3D:	$1/m_W(T) \quad \gg a \gg 1/(m_H(T)L_x)$

  Our largest $a = \frac{9}{5}\frac{1}{T}$
- *Superrenormalizable*: $g \to 0$ limit unique, no $\Lambda_{\rm LATT}$ $\to$ comparison with P.T. in 3D (and 4D!) straightforward
- Number of variables is much less in 3D than in 4D.

4D continuum SU(2) $- \phi$

↓ 2–loop matching in P.T.

3D continuum SU(2) $- \phi$, (A_0)

↓ 2–loop, unique at $g \to 0$

3D lattice SU(2) $- \phi$

Lattice action ($\Phi^2 \equiv \frac{1}{2}\mathrm{Tr}\,\phi^\dagger\phi$):

$$S = \beta_G \sum_\Box (1 - \frac{1}{2}\mathrm{Tr}\,U_\Box) - \beta_H \sum_{x,i} \frac{1}{2}\mathrm{Tr}\,\phi_x^\dagger U_{x,i}\phi_{x+i} + \sum_x [\Phi^2 + \beta_R(\Phi^2 - 1)^2]$$

Exact relation between continuum and lattice parameters
$(g, T, \lambda, m^2) \longleftrightarrow (\beta_G, \beta_H, \beta_R, a)$

$$\beta_G = \frac{4}{g^2 aT}$$

Our strategy:

- Fix $g = 2/3$, $m_W = 80.6$ GeV, and $m_H = 35$, 60 or 70 GeV. This gives $\lambda = 1/8g^2(m_H/m_W)^2$.
- Use $\beta_G = 5, 8, 12, 20 \longrightarrow aT = \frac{9}{5}, \frac{9}{8}, \frac{9}{12}, \frac{9}{20}$.
- Adjust $T \rightarrow (\beta_H, \beta_R)$ until transition found.
- For fixed β_G, we extrapolate measurements to the thermodynamic limit $V \rightarrow \infty$. The results are extrapolated further to the continuum limit $\beta_G \rightarrow \infty$.

m_H	β_G	volumes
35	8	$6^2 \times 18$, $8^2 \times 24$, $10^2 \times 30$, $12^2 \times 36$, $14^2 \times 42$, $8^2 \times 80$, $10^2 \times 80$
	12	12^3, $12^2 \times 24$, $12^2 \times 48$, 16^3, $16^2 \times 32$, $18^2 \times 36$, $20^2 \times 40$, $22^2 \times 44$
	20	10^3, $10^2 \times 30$, $12^2 \times 36$, $16^2 \times 48$, $20^2 \times 60$, $24^2 \times 72$
60	5	$12^2 \times 72$, $16^2 \times 80$
	8	12^3, 16^3, 24^3, 32^3, $20^2 \times 140$, $24^2 \times 120$, $30^2 \times 120$
	12	16^3, 24^3, 32^3, 40^3, $26^2 \times 156$, $30^2 \times 150$, $36^2 \times 144$
	20	16^3, 24^3, 32^3, 40^3
70	8	12^3, 16^3, 24^3, 32^3
	12	12^3, 16^3, 24^3, 32^3, 40^3, 48^3
	20	12^3, 16^3, 24^3, 32^3, 40^3, 48^3

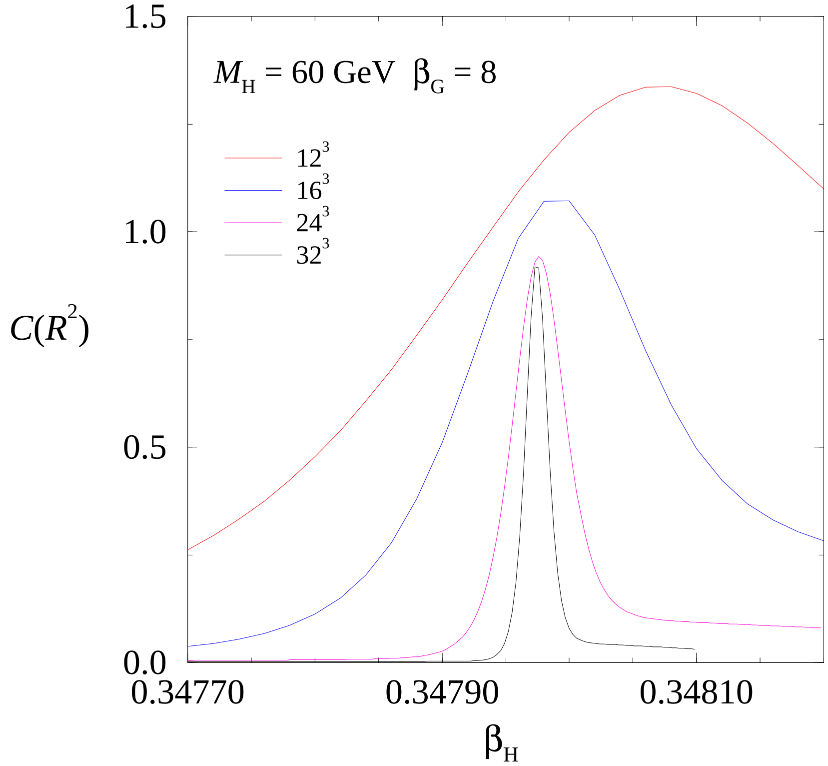
- Red volumes: "flat" histograms; surface tension measurements are possible.
- Total # lattices: 57.
- Computers: Cray C90, Cray X-MP, HP7000, IBM RS6000

Transition temperature $\beta_{H,c} \longrightarrow T_c$ is determined with 5 methods:

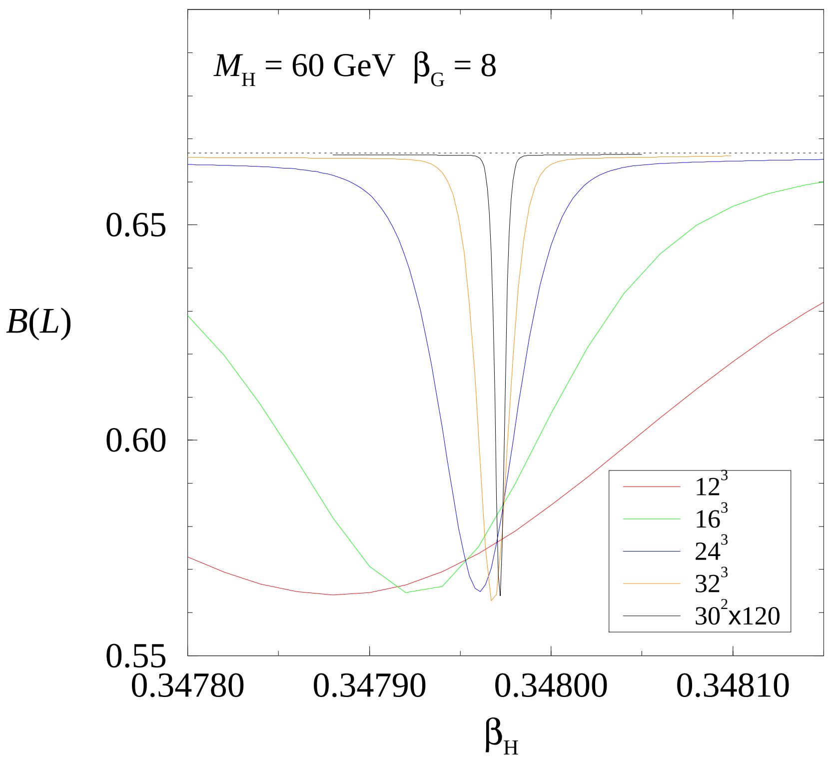
1) Maximum of $R^2 = \frac{1}{2}\mathrm{Tr}\,\phi^\dagger\phi$ susceptibility
2) Maximum of $L = \frac{1}{2}\mathrm{Tr}\,V_i^\dagger U_{ij} V_j$ susceptibility, ($\phi = R \times V$)
3) Minimum of the Binder cumulant of L
4) Equal weight values for the distributions of R^2
5) Equal height value for $p(L)$

All methods converge in the limit $V \rightarrow \infty$. However, we keep the $T_{c,V=\infty}$ values separate and extrapolate with each method to $\beta_G \rightarrow \infty$.

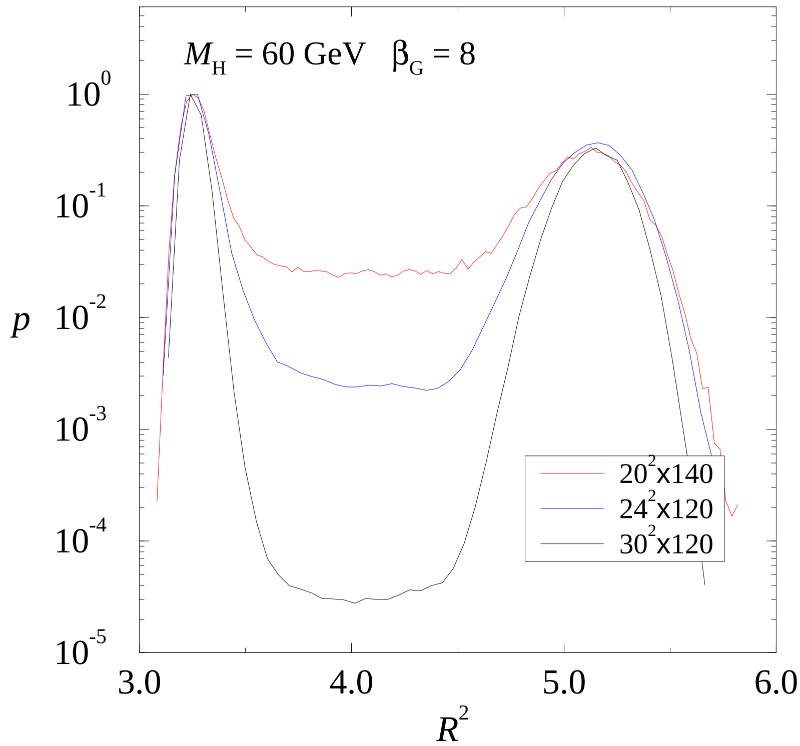
m_H	$T_c^{\rm pert}$	$T_c^{\rm latt}$
35	93.3	92.64(7)
60	140.1	138.20(4)
70	157.0	154.5(1)



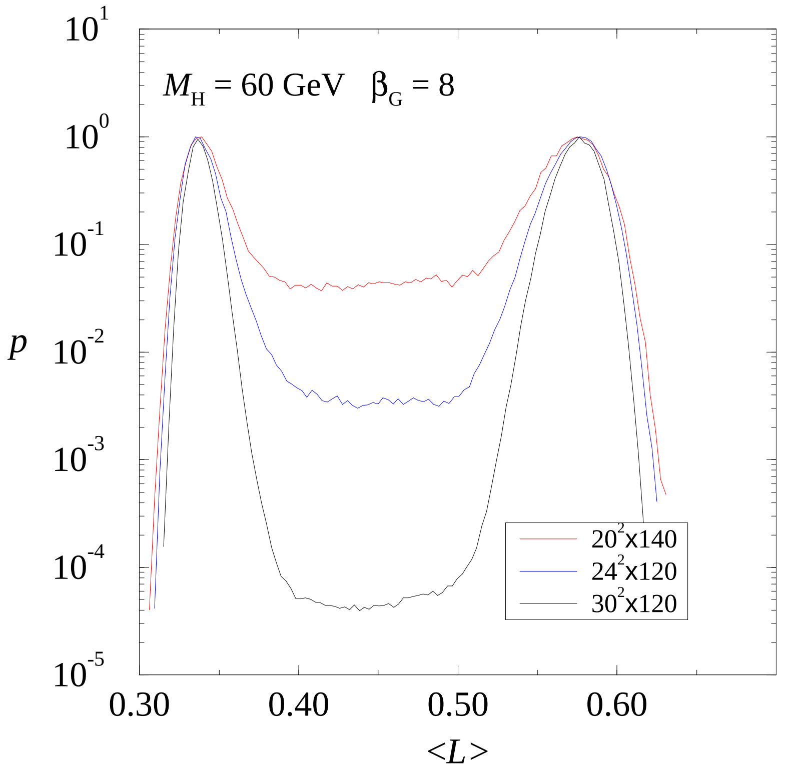
M_H = 60 GeV β_G = 8
12^3
16^3
24^3
32^3
$C(R^2)$
1.5
1.0
0.5
0.0
0.34770
0.34790
0.34810
β_H



M_H = 60 GeV β_G = 8
$B(L)$
0.65
0.60
0.55
12^3
16^3
24^3
32^3
30^2x120
0.34780
0.34790
0.34800
0.34810
β_H

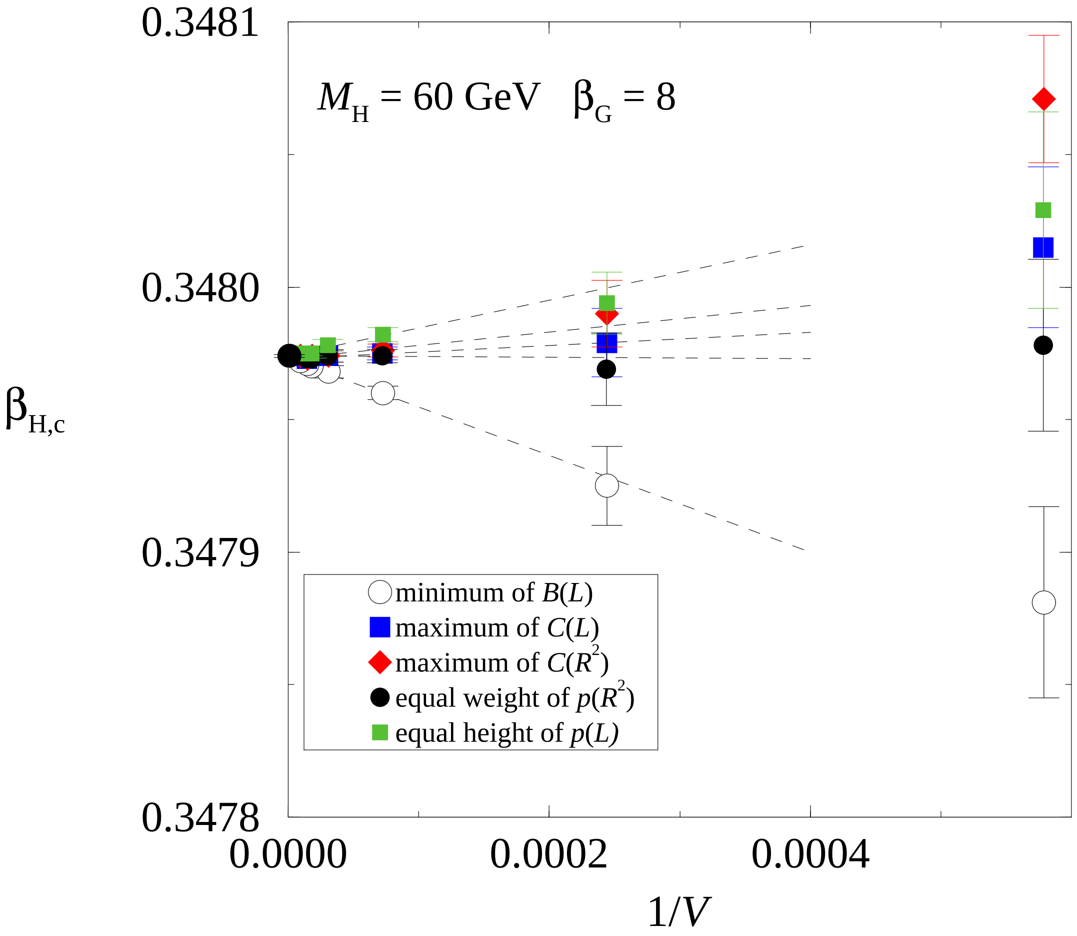


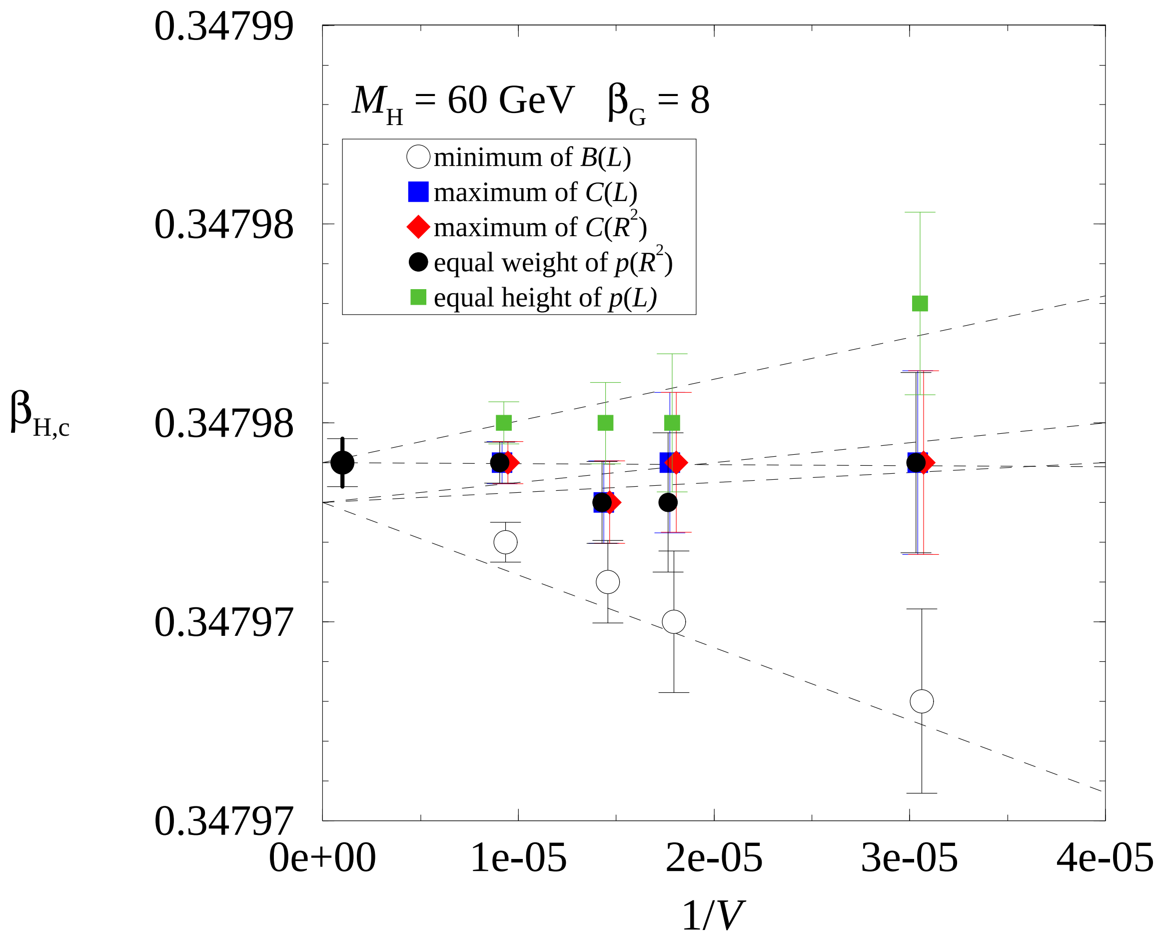
M_H = 60 GeV β$_G$ = 8
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
p
3.0
4.0
5.0
6.0
R^2
20^2x140
24^2x120
30^2x120



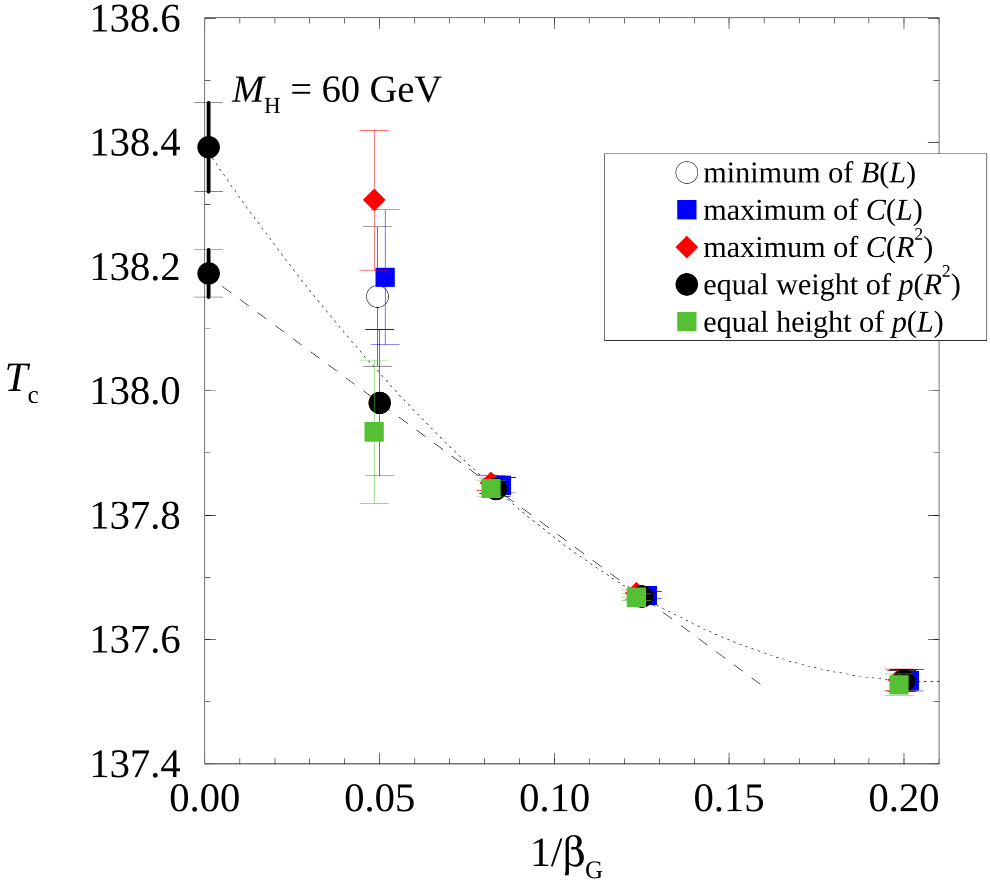
M_H = 60 GeV β$_G$ = 8
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
p
0.30
0.40
0.50
0.60
<L>
20^2x140
24^2x120
30^2x120

Extrapolation to the thermodynamic limit $V \longrightarrow \infty$

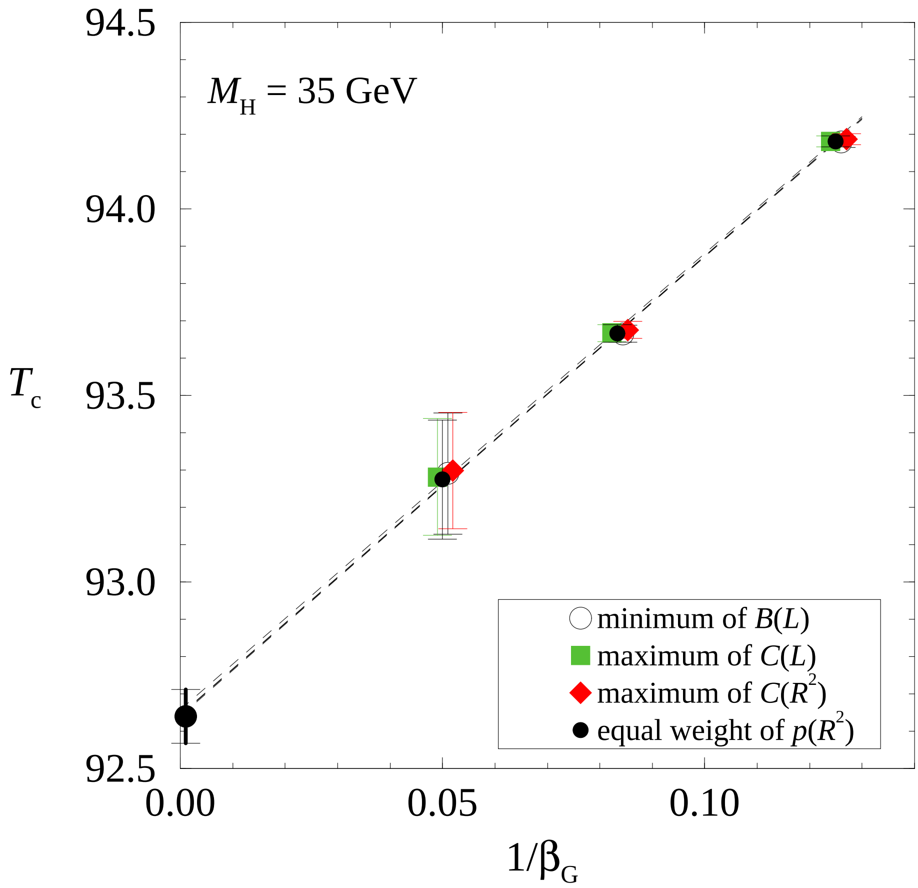



M_H = 60 GeV β_G = 8
minimum of $B(L)$
maximum of $C(L)$
maximum of $C(R^2)$
equal weight of $p(R^2)$
equal height of $p(L)$
$\beta_{H,c}$
0.34799
0.34798
0.34798
0.34797
0.34797
0e+00
1e-05
2e-05
3e-05
4e-05
$1/V$

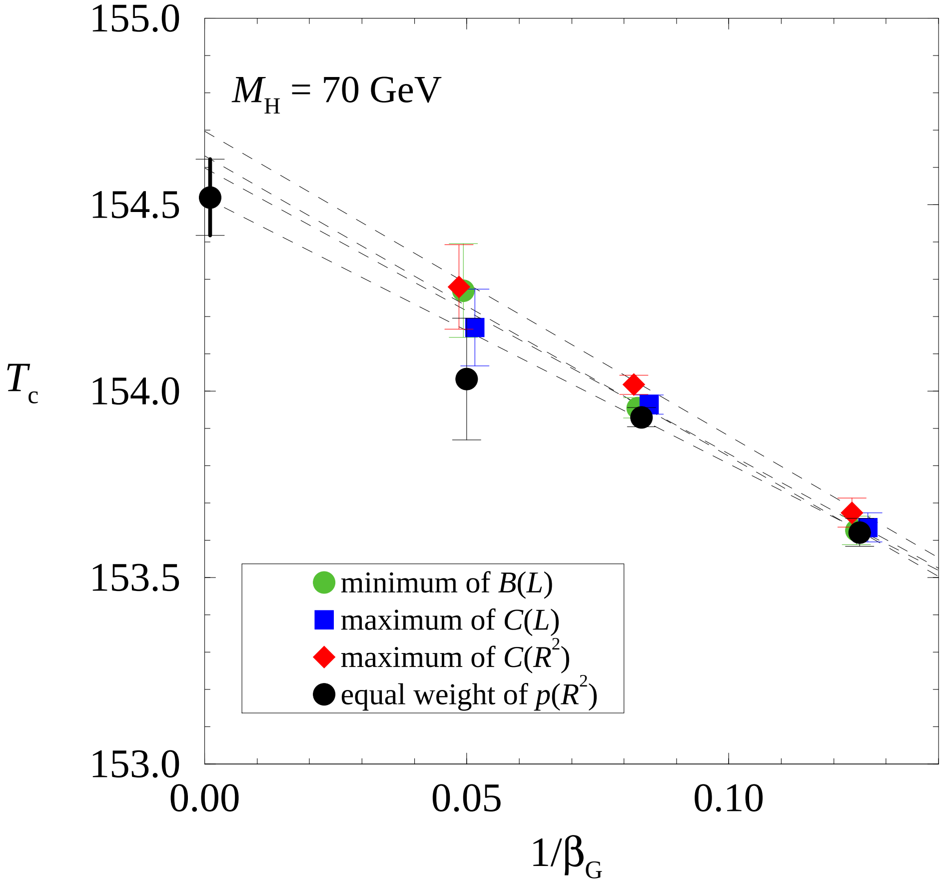
$m_H = 60$ GeV extrapolation to the continuum limit
$T_c^{\text{pert}} = 140.1$ GeV



$m_H = 35$ GeV extrapolation to the continuum limit
$T_c^{\rm pert} = 93.3$ GeV



$m_H = 70$ GeV extrapolation to the continuum limit
$T_c^{\rm pert} = 157.0$ GeV



Latent heat L:

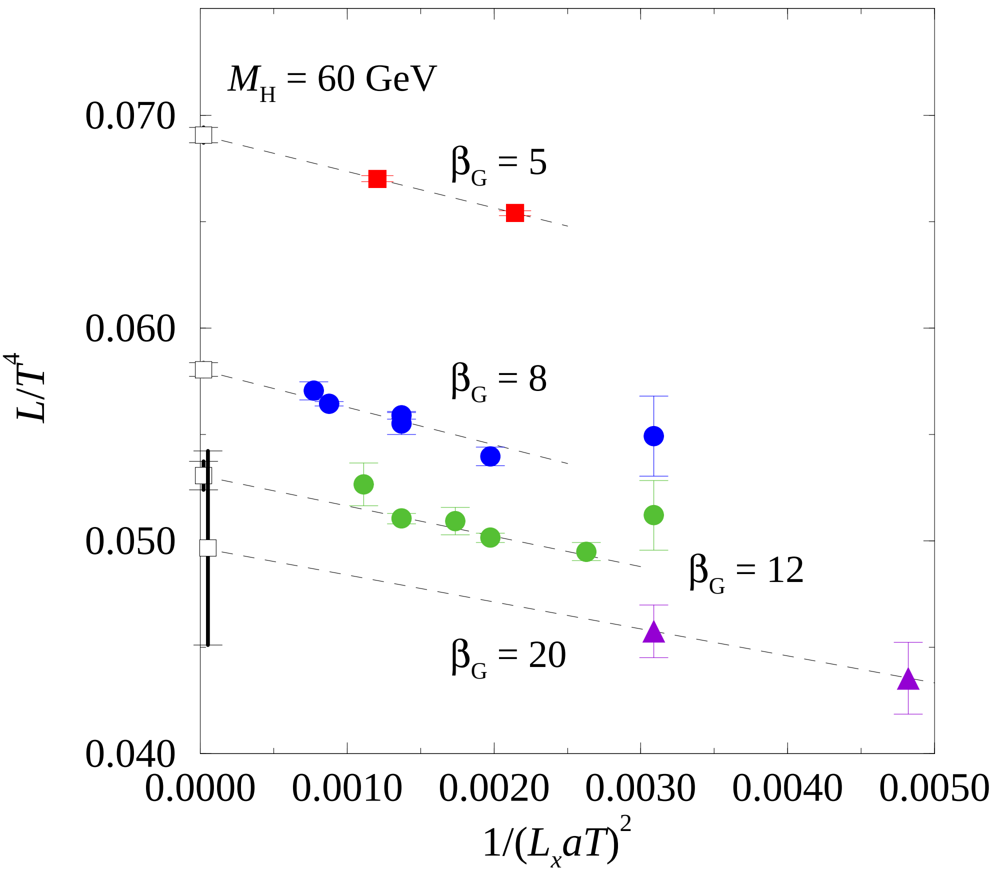
$$\frac{L}{T^4} = \frac{1}{T^3}\frac{d}{dT}\Delta p = \frac{1}{VT^3}\frac{d}{dT}\Delta \log Z = \frac{1}{VT^3}\frac{d}{dT}\Delta P$$

(Δp pressure difference, ΔP probability)

$v(T)/T$ is calculated from ΔR^2

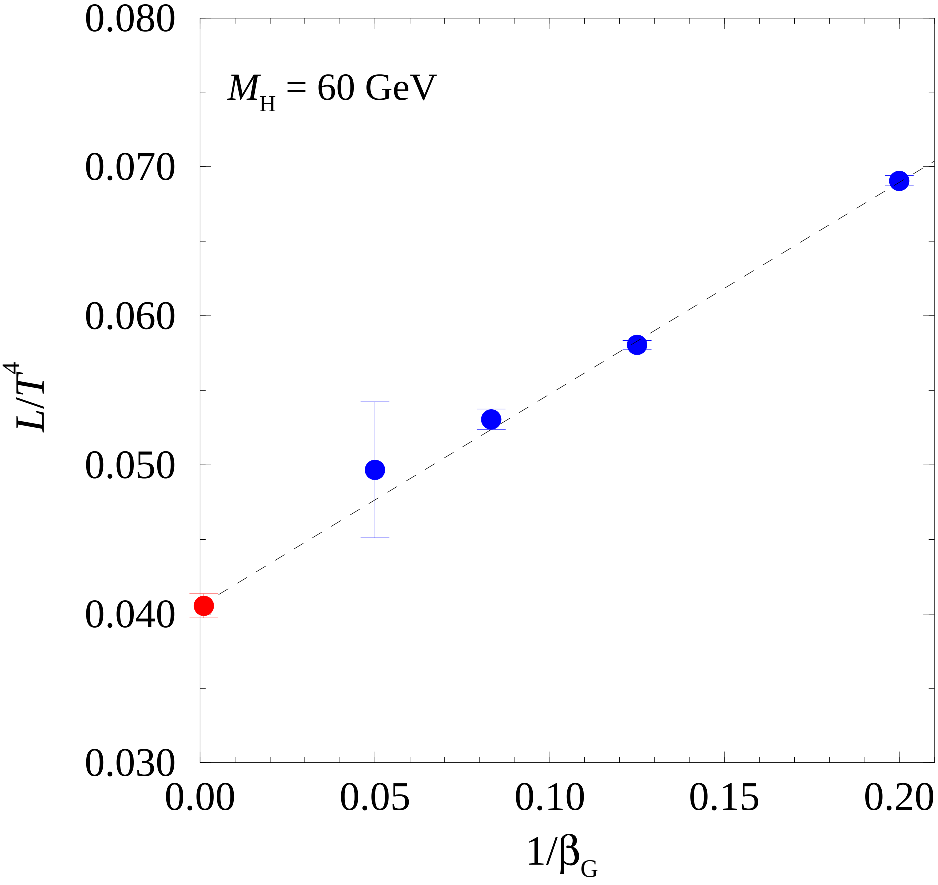
Both of the above quantities require good separation of the two phases

m_H	L^{pert}/T_c^4	L^{latt}/T_c^4	$v^{\text{pert}}(T_c)/T_c$	$v^{\text{latt}}(T_c)/T_c$
35	0.22	0.256(10)	1.75	1.86(3)
60	0.041	0.0405(8)	0.68	0.691(7)
70	0.028	0.0273(16)	0.55	0.57(2)



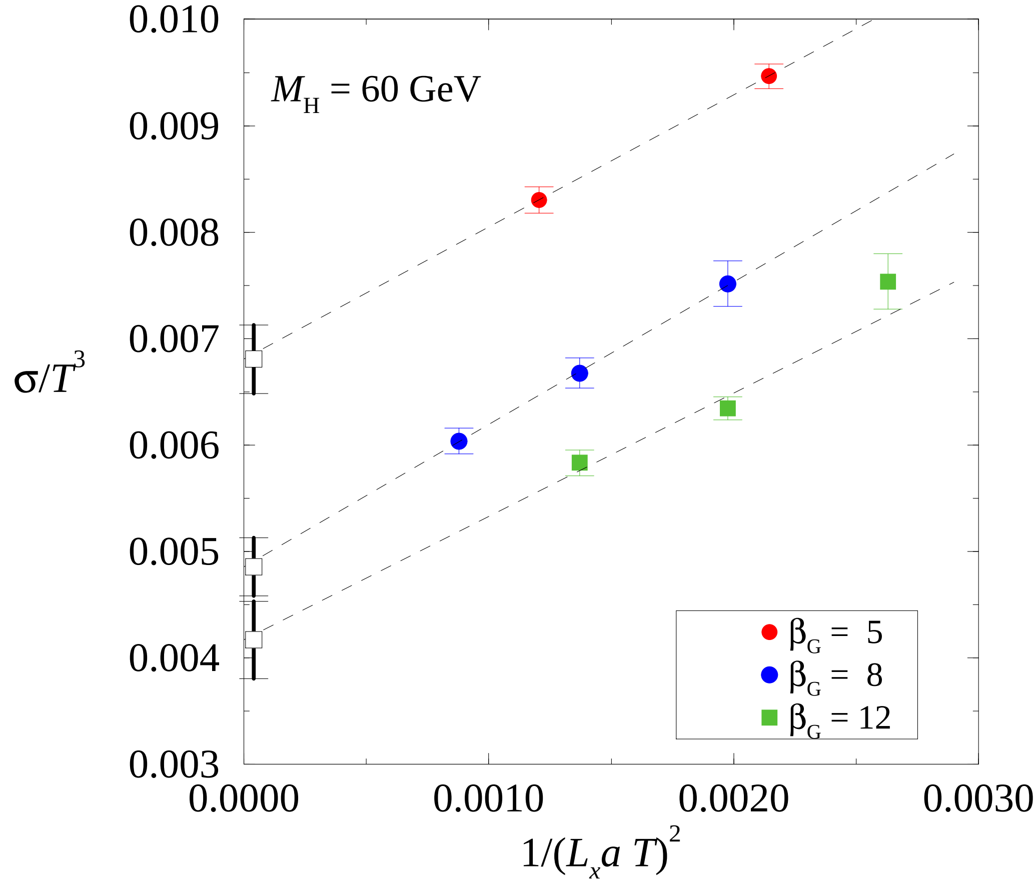
M_H = 60 GeV
β_G = 5
β_G = 8
β_G = 12
β_G = 20
L/T^4
$1/(L_x aT)^2$
0.070
0.060
0.050
0.040
0.0000
0.0010
0.0020
0.0030
0.0040
0.0050

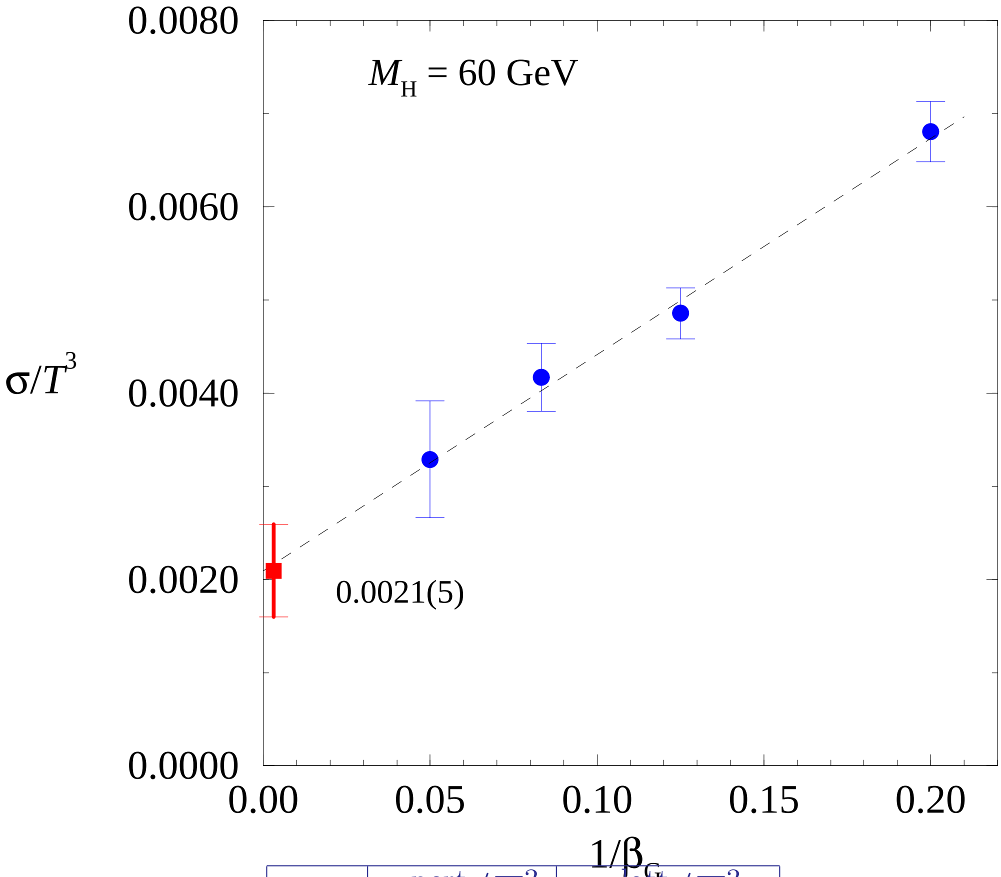
$m_H = 60$ GeV $L^{\text{pert}}/T_c^4 = 0.041$



Surface tension σ/T^3: on a $L_x^2 \times L_z$ volume

$$\sigma/T = \frac{1}{2L_x^2} \log \frac{P_{\rm max}}{P_{\rm min}} + \frac{1}{L_x^2}[\frac{3}{4} \log L_z - \frac{1}{2} \log L_x + const]$$



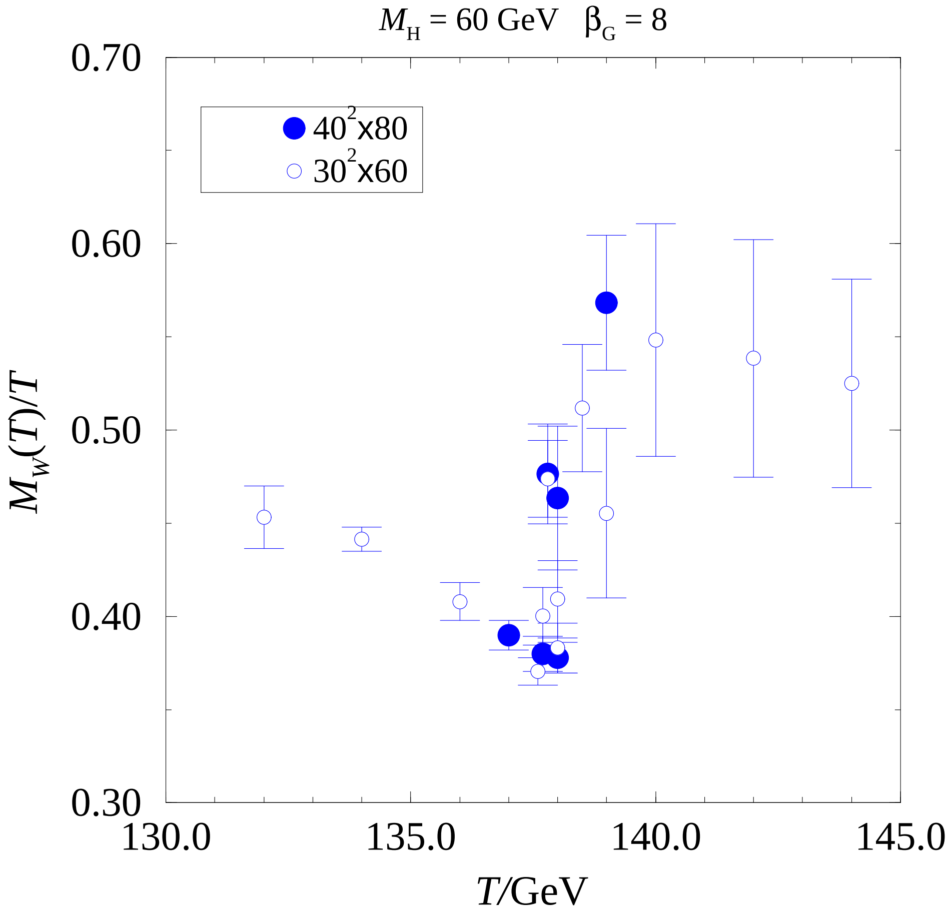


m_H	$\sigma^{\text{pert}}/T_c^3$	$\sigma^{\text{latt}}/T_c^3$
35	0.061	0.0917(25)
60	0.008	0.0021(5)
70	0.005	–

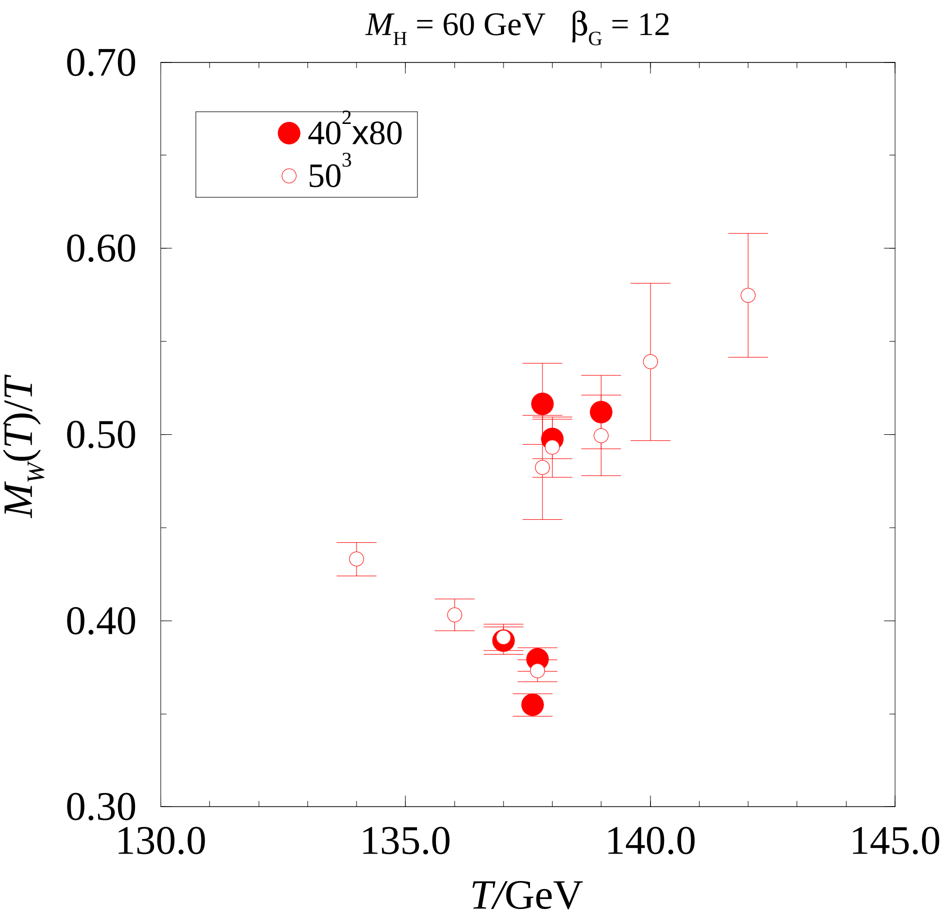
$m_H = 35$ GeV has only $\beta_G = 8$ – no continuum limit possible

Conclusions:

- 3D DR-theory is a *very* powerful tool for studying the EW transition.
- EW phase transition is of first order $m_H \lesssim 70$ GeV.
- 2-loop P.T. provides a good guideline for the transition. However, deviations are clearly seen.
- $T_c^{\rm latt} < T_c^{\rm pert}$ (difference $\sim 1.5\%$).
- $\sigma^{\rm latt} < \sigma^{\rm pert}$ at $m_H = 60$ GeV.



M_H = 60 GeV β_G = 8
0.70
0.60
0.50
0.40
0.30
40^2x80
30^2x60
$M_W(T)/T$
130.0
135.0
140.0
145.0
T/GeV


M_H = 60 GeV β_G = 12
0.70
0.60
0.50
0.40
0.30
130.0
135.0
140.0
145.0
T/GeV
M_W(T)/T
40^2x80
50^3