Supersymmetry, autumn 2010, exercise 1 (please return on Sep 16)

1. From the SLAC QSPIRES database, find the number of citations for
   Coleman and Mandula, PR 159 (1967) 1251.
   Neveu and Schwartz, NPB31 (1971) 86; PRD4 (1971) 1109.
   Ramond, PRD3 (1971) 2415.
   Volkov and Akulov, PLB 46 (1973) 109.
   Golfand and Likhtman, JETP Letters 13 (1971) 323.
   Wess and Zumino, NPB70 (1974) 39; PLB49 (1974) 52.

2. In the world sheet, light-cone coordinates $\sigma^{\pm} = \tau \pm \sigma$ can be defined. Here τ is the time coordinate, and σ labels the string. Space-time coordinates are denoted by $X^{\mu}(\sigma, \tau)$, and $\psi_i^{\mu}(\sigma, \tau)$, $i = 1, 2$ are fermionic field doublets. Show that the Gervais-Sakita action (T is string tension, and $\mu = 0, 1, \ldots, d-1$)

$$I[X, \phi] = \int d\sigma^+ \int d\sigma^- \left[T \frac{\partial X^{\mu}}{\partial \sigma^+} \frac{\partial X_{\mu}}{\partial \sigma^-} + i\psi_2^{\mu} \frac{\partial}{\partial \sigma^+} \psi_{2\mu} + i\psi_1^{\mu} \frac{\partial}{\partial \sigma^-} \psi_{1\mu} \right]$$

   is invariant under the worldsheet supersymmetry transformation

$$\begin{aligned}
\delta\psi_2^{\mu}(\sigma^+, \sigma^-) &= iT\alpha_2(\sigma^-) \frac{\partial}{\partial \sigma^-} X^{\mu}(\sigma^+\sigma^-), \\
\delta\psi_1^{\mu}(\sigma^+, \sigma^-) &= iT\alpha_1(\sigma^+) \frac{\partial}{\partial \sigma^+} X^{\mu}(\sigma^+\sigma^-), \\
\delta X^{\mu}(\sigma^+\sigma^-) &= \alpha_2(\sigma^-)\psi_2^{\mu}(\sigma^+, \sigma^-) + \alpha_1(\sigma^+)\psi_1^{\mu}(\sigma^+, \sigma^-),
\end{aligned}$$

   where α_1, α_2 are infinitesimal fermionic functions of σ^+ and σ^-.

3. Using Λ as a physical cut-off for momentum integration, calculate the fermion self-energy $\delta m_F = \Sigma_F(p)|_{p=m_F}$, where

$$-i\Sigma_F(p) = \left(\frac{-i\lambda_F}{\sqrt{2}} \right)^2 i^2 \int \frac{d^4k}{(2\pi)^4} \frac{(\not{k} + m_F)}{(k^2 - m_F^2)[(k-p)^2 - m_S^2]}.$$

   (Hint. see discussion in S. Dawson, hep-ph/9712464)