

# Supersymmetry

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**Lectures:** Wed 14-16 A315

**Exercises:** Thu 14-16 (?)

Problem solutions should be returned by Wednesday (?) at noon to the box located in the A-wing of the 2nd floor.

**20 % of the grade.**

## Literature:

### Lecture notes

**Binetruy:** *Supersymmetry* (Oxford University Press, 2006);

**Bailin, Love:** *Supersymmetric Gauge Field Theory and String Theory* (IoP, Bristol UK, 1994) Chapters 1-6;

Baer, Tata: *Weak Scale Supersymmetry* (Cambridge University Press, 2006);

Drees, Godbole, Roy: *Theory and phenomenology of sparticles* (World Scientific, Singapore, 2004);

Weinberg: *The quantum theory of fields, vol III, Supersymmetry* (Cambridge University Press, 2000);

Wess, Bagger: *Supersymmetry and Supergravity* (Princeton University Press, Princeton NJ, 1992);

Martin: *A Supersymmetry Primer*, hep-ph/9709356, in G. Kane, *Perspectives on Supersymmetry*, World Scientific, 1998.

## Outline:

### Introduction

- History and motivation
- Dirac, Majorana, and Weyl fermions
- SUSY algebra
- SUSY multiplets

### Simplest SUSY model:

- Wess-Zumino model

### Superfield formulation of SUSY

- Chiral superfields

### Supersymmetric gauge theories

- Vector superfields

### Spontaneous breaking mechanisms of SUSY

- O'Raifeartaigh models
- Fayet-Iliopoulos term

The minimal supersymmetric standard model (MSSM)

The MSSM Higgs sector

Sparticles at colliders

Dark matter

Flavour changing neutral currents (FCNC)

Supersymmetry breaking in the hidden sector

Gravity mediated SUSY breaking

Gauge mediated symmetry breaking

Anomaly mediated symmetry breaking

More phenomenology if time allows...

**Supersymmetry** relates particles with integer spin to particles with half integer spin in relativistic quantum field theory.

### **Some history:**

#### **A no go theorem:**

The most general Lie algebra, commuting with S-matrix, consists of the Poincare group ( $P_\mu, J_{\mu\nu}$ ) and internal symmetry (independent of spin and momentum) generators.



Coleman and Mandula, PRD 159 (1967) 1251.

#### **Supersymmetry in string theories:**

Theory on 2-dimensional world sheet with bosonic and fermionic fields under appropriate boundary conditions exhibit supersymmetry.

Neveu, Schwartz, NPB 31 (1971) 86; Ramond, PRD 3 (1971) 2415; Gervais, Sakita, NPB 34 (1971) 632.

## Supersymmetry in 4-dimensional space-time:

Poincare group  superalgebra  SUSY field theories

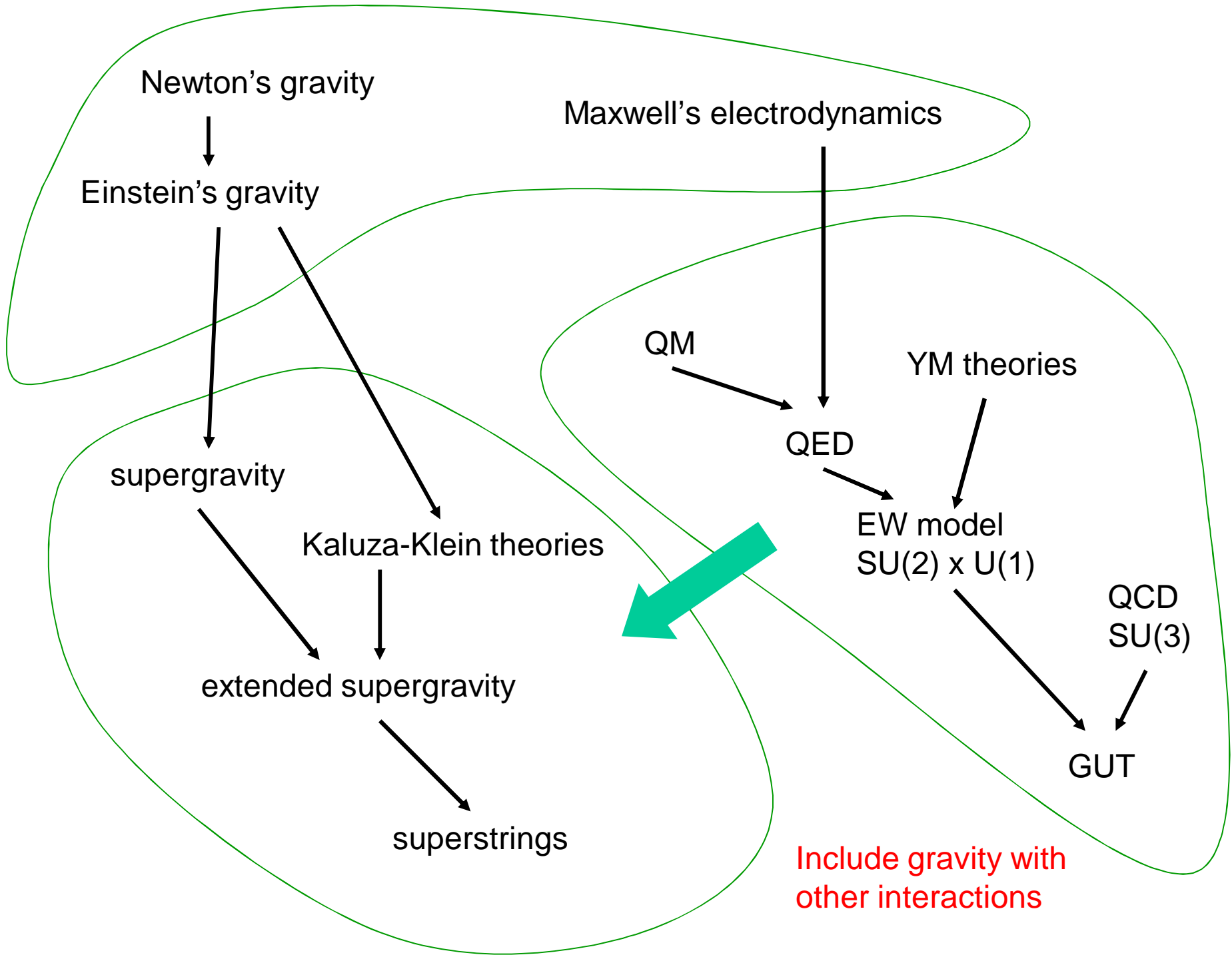
Golfand, Likhtman, JETP Letters 13 (1971) 323.

Spontaneously broken supersymmetry  
(Goldstone fermion=neutrino **Not true**)

Volkov, Akulov, PLB 46 (1973) 109.

Examples of supersymmetric models

Wess, Zumino, NPB 70 (1974) 39; PLB 49 (1974) 52.



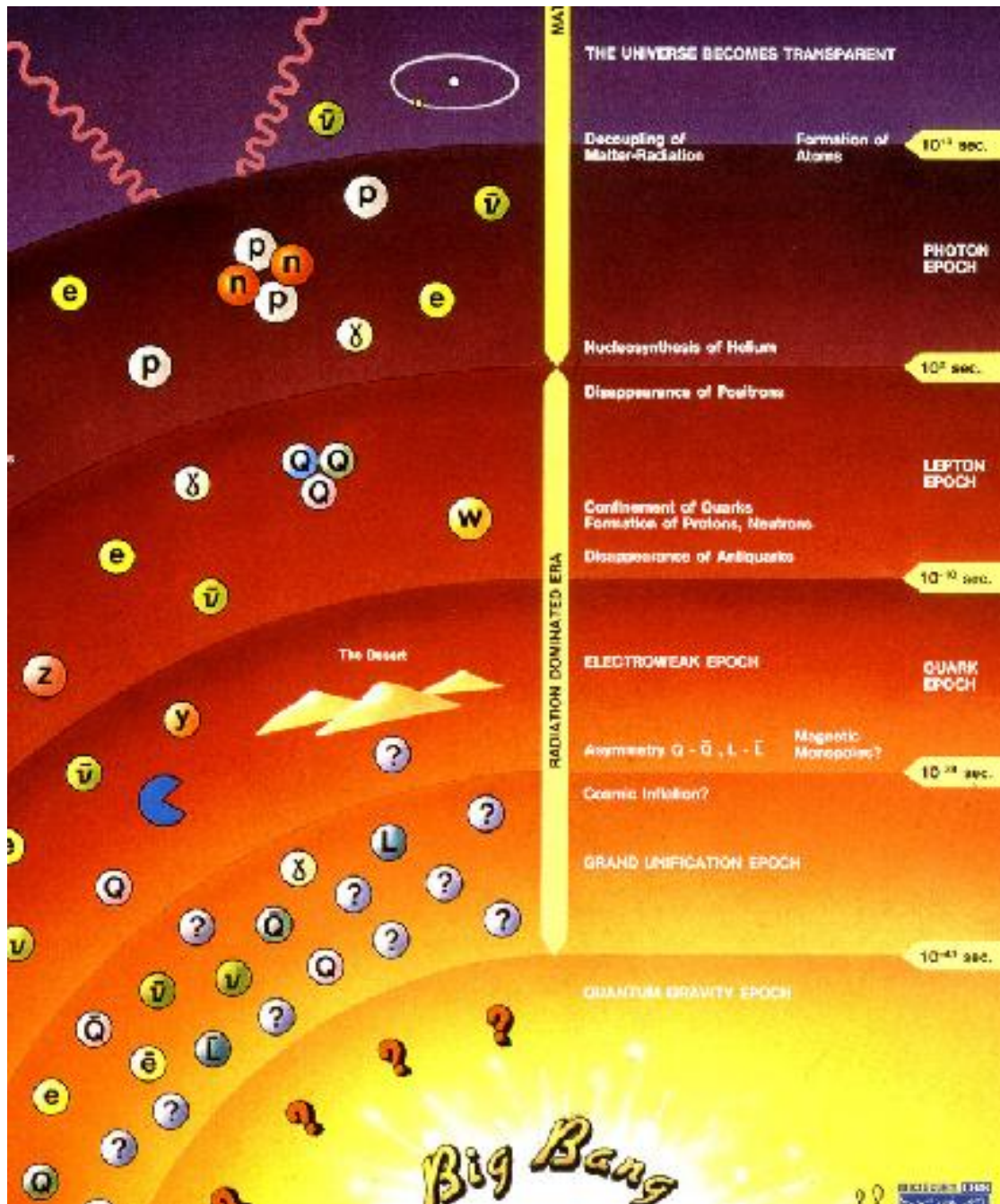
## Motivation: why SUSY?

**Electromagnetism** Quantum Electro Dynamics,  $U(1)_{em}$   
Force carrier: photon  $\gamma$ , massless, spin=1  
Coupling to **charged** matter particles, such as e, u, d  
Strength of the coupling  $\alpha_{em}=1/137$ .

**Weak interactions**  $SU(2)_L \times U(1)_Y/U(1)_{em}$   
Force carriers  **$W^+, W^-, Z$**  bosons, massive, spin=1  
Coupling to particles with **weak** charges.  
Strength of the coupling  $\alpha_w(M_Z) \cong 1/30$ .  
Observed weakness due to the massiveness of W, Z:  
 $M_W, M_Z \cong 100 \text{ GeV}$

**Strong interactions** Quantum Chromodynamics,  $SU(3)_c$   
Force carriers: 8 massless gluons,  $g^a$ , spin=1  
Coupling to **coloured** particles, such as u, d  
Strength of the coupling  $\alpha_s(M_Z) \cong 1/10$ .

**Gravity** Quantum gravity (?):  
No known self-consistent quantum theory; superstrings,  
extra dimensions, ...  
Force carrier: massless gravitons, spin=2  
 $10^{-40}$  weaker than electromagnetism



300000 years

1 Å  
10 eV

Magnetism,  
electricity,  
weak theory,  
QCD, gravity

100 s

1 fm  
1 MeV

QED,  
weak theory,  
QCD, gravity

10<sup>-10</sup> s

10<sup>-19</sup> m  
100 GeV

electroweak  
model,  
QCD, gravity

10<sup>-32</sup> s

10<sup>-32</sup> m  
10<sup>16</sup> GeV

SUSY?,  
GUT?,  
gravity

10<sup>-43</sup> s

10<sup>-35</sup> m  
10<sup>19</sup> GeV

SUGRA?,  
GUT?,  
STRINGS?

## The Standard Model (SM) of particle physics:

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

How to give masses for W, Z bosons and matter?

Why are the masses stable under quantum correction between the weak scale and GUT scale ( $10^{16}$  GeV)?

Hierarchy problem (naturalness problem):  $M_W/M_{\text{Planck}} \cong 10^{-17}$

Why a problem?

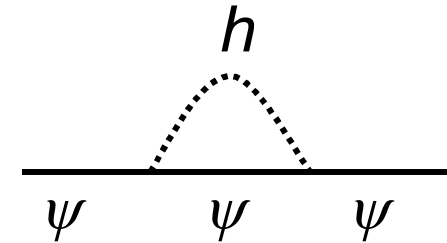
Let us consider radiative corrections to particle masses.

**Photon** mass terms  $m_\gamma^2 A^\mu A_\mu$  break gauge invariance

→  $m_\gamma = 0$

**Fermions** Consider a single fermion coupled to a massive Higgs scalar  $\phi$ :

$$L_\phi = \bar{\psi} i \gamma^\mu \partial_\mu \psi + |\partial_\mu \phi|^2 - m_S^2 |\phi|^2 - \left( \frac{\lambda_F}{2} \bar{\psi} \psi \phi + h.c. \right);$$



$$\phi = \frac{1}{\sqrt{2}} (h + v); \quad m_F = \frac{1}{\sqrt{2}} \lambda_F v;$$

$$-i \Sigma_F(p) = \left( \frac{-i \lambda_F}{\sqrt{2}} \right)^2 i^2 \int \frac{d^4 k}{(2\pi)^4} \frac{k + m_F}{(k^2 - m_F^2)((k - p)^2 - m_S^2)}$$

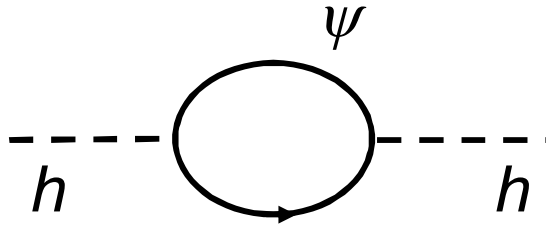
$$m_F^r = m_F + \delta m_F = m_F + \Sigma_F(p) \Big|_{p=m_F};$$

$$\delta m_F = -\frac{3 \lambda_F^2 m_F}{64 \pi^2} \ln \frac{\Lambda^2}{m_F^2} + \dots \propto m_F$$

If  $m_F \rightarrow 0$ ,  $L$  has a chiral symmetry  $\psi_{L,R} \rightarrow e^{i\theta_{L,R}} \psi_{L,R}$

This symmetry is broken by the Yukawa coupling. Thus the corrections are proportional to  $m_F$ .

Scalars:



$$-i \Sigma_S(p^2) = \left( \frac{-i\lambda_F}{\sqrt{2}} \right)^2 (-1) i^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(k + m_F)[(k - p) + m_F]}{(k^2 - m_F^2)((k - p)^2 - m_F^2)}$$

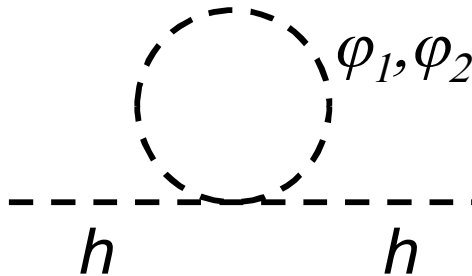


$$\delta m_h^2 = -\frac{\lambda_F^2}{8\pi^2} \left[ \Lambda^2 + (m_S^2 - 6m_F^2) \ln \frac{\Lambda^2}{m_F^2} + \dots \right]$$

Quadratic divergence, which is independent of the Higgs mass (setting Higgs mass equal to zero does not increase the symmetry of  $L$ ).

Take two complex scalar fields, interacting with  $h$  :

$$L_{\Phi} = \left| \partial_{\mu} \varphi_1 \right|^2 + \left| \partial_{\mu} \varphi_2 \right|^2 - m_{S_1}^2 |\varphi_1|^2 - m_{S_2}^2 |\varphi_2|^2 + \lambda_S |\phi|^2 \left( |\varphi_1|^2 + |\varphi_2|^2 \right) + L_{\Phi};$$

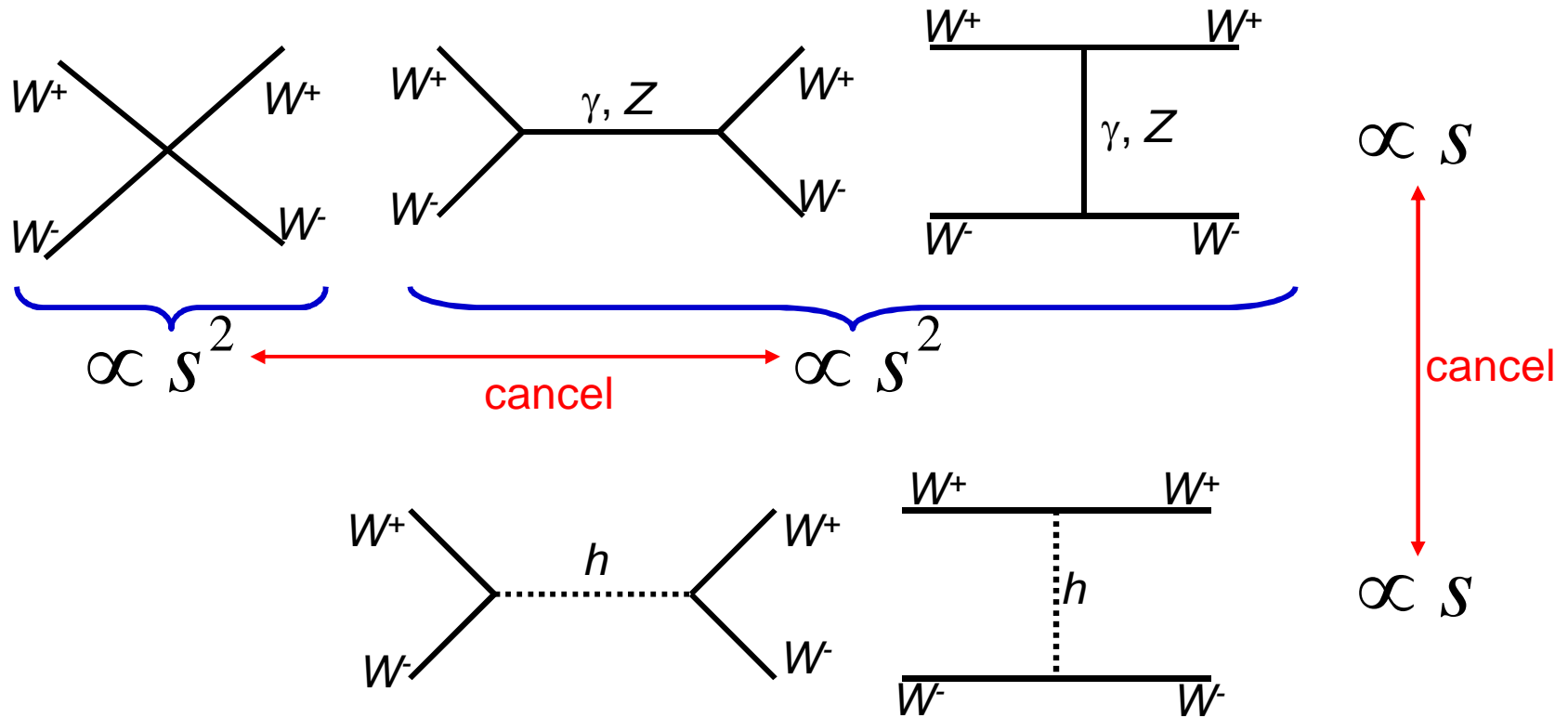


$$\delta m_h^2 = \frac{\lambda_S}{8\pi^2} [\Lambda^2 + \dots]$$

Thus if  $\lambda_F^2 = \lambda_S$ , the quadratic divergencies would cancel. Note that one could do this with top quark and Higgs. No *symmetry* enforces this cancellation, thus the approach would fail in two loops.

Other loop corrections exist, but they do not have quadratic divergencies.

# What is known about the Higgs boson mass?

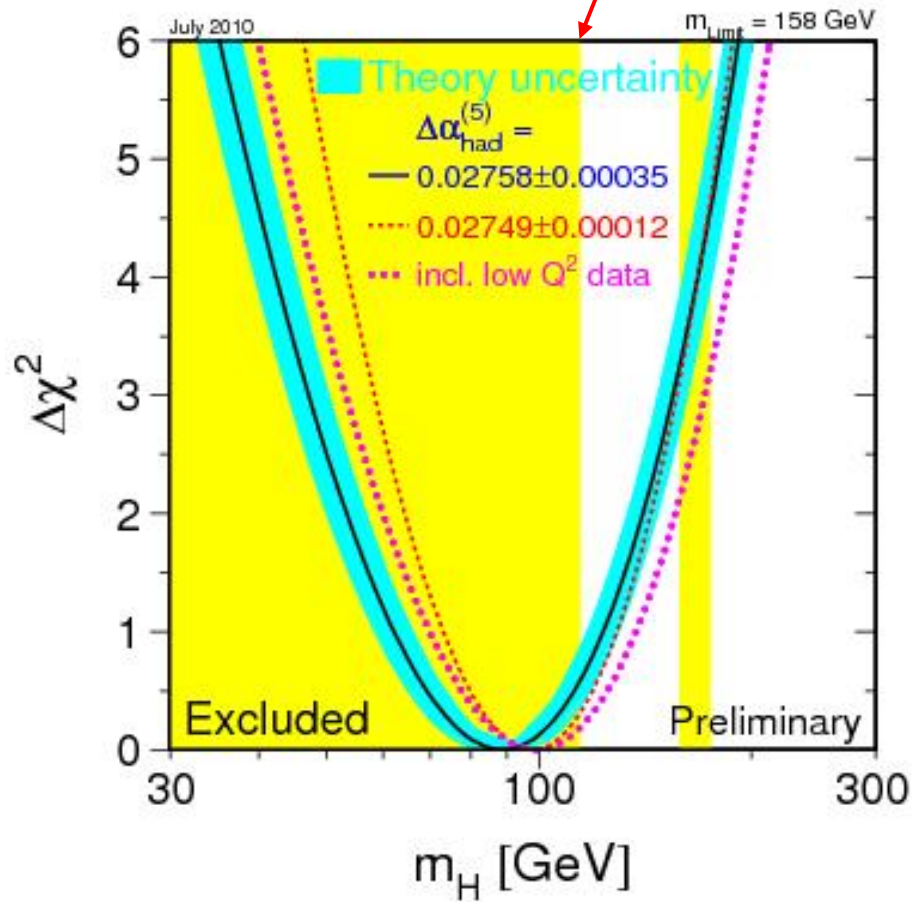


From the partial wave unitarity, it is theoretically expected that

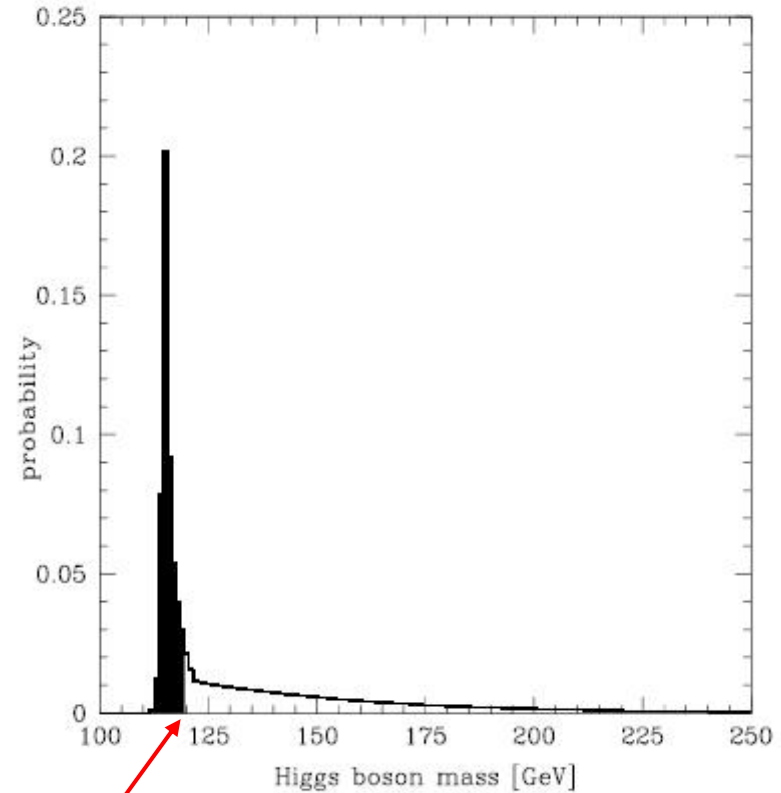
$$m_{\text{Higgs}} < 870 \text{ GeV}$$

LEP:  $M_H > 114.4$  GeV

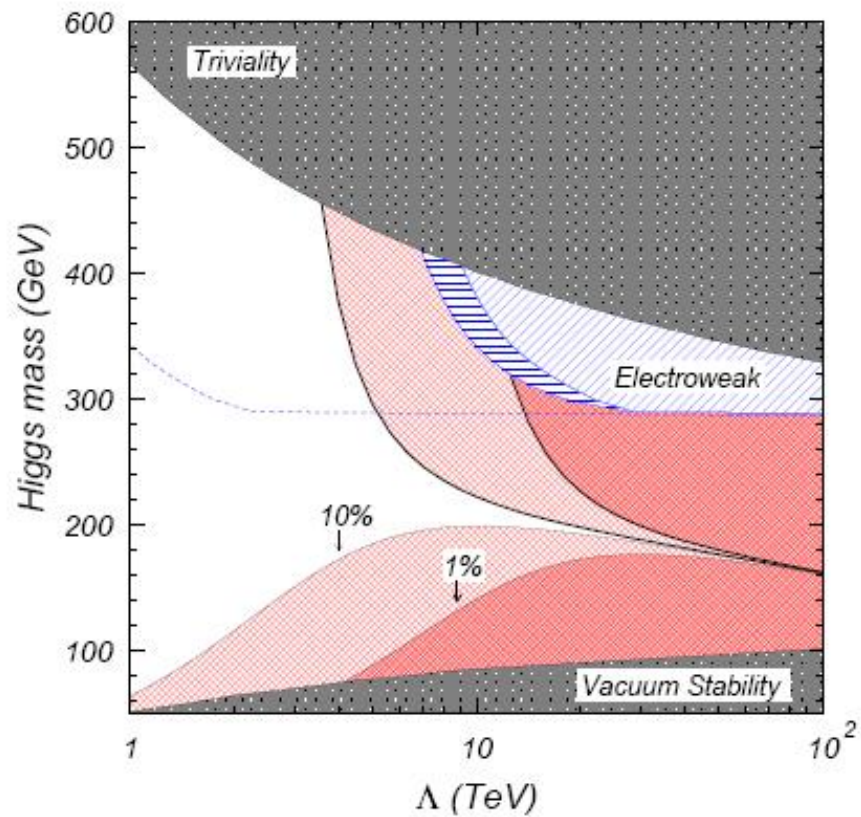
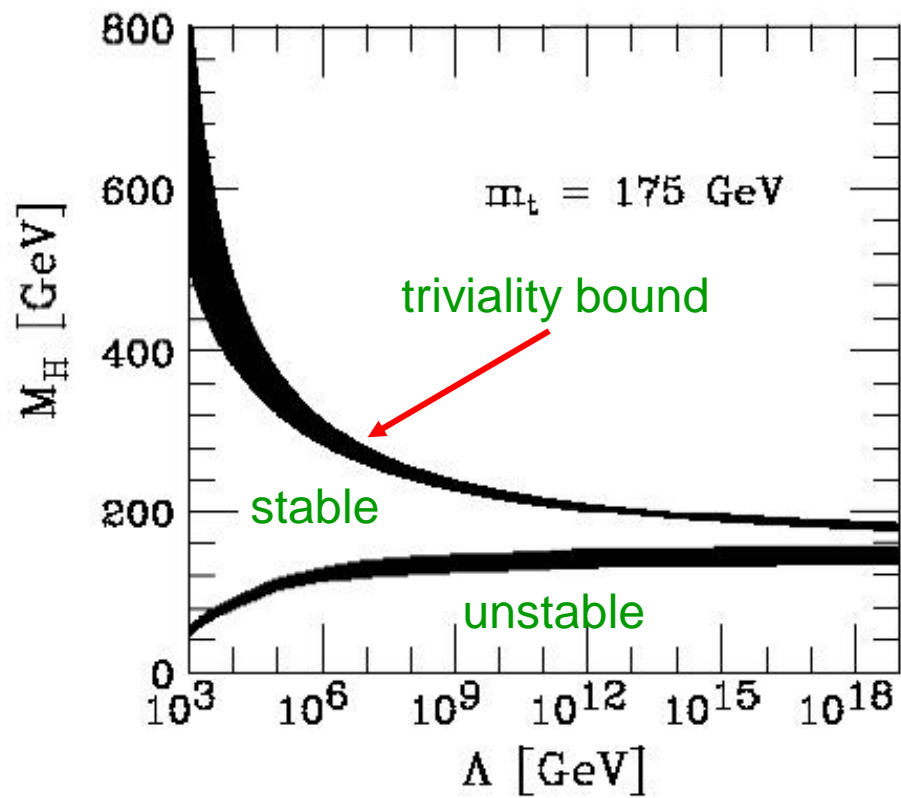
Erlar, PRD 63 (2001) 071301.



$$M_H = 89^{+35}_{-26} \text{ GeV}$$



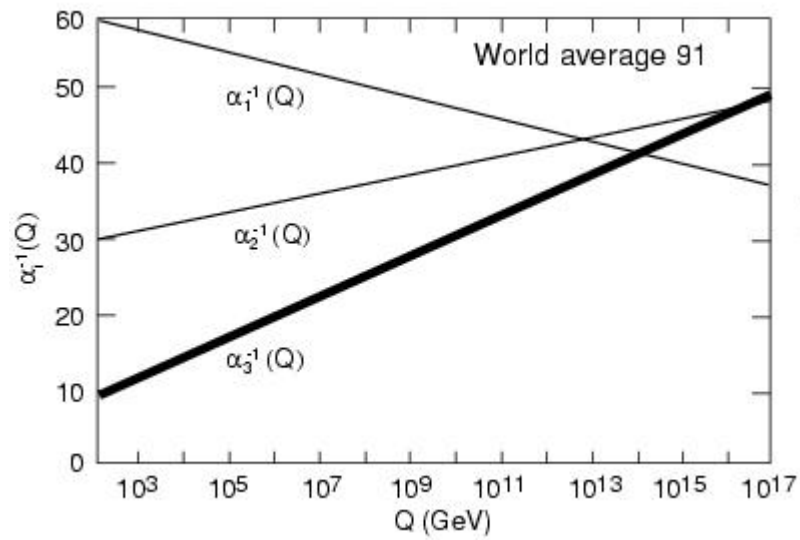
50 % probability to have  $m_h$  below or above



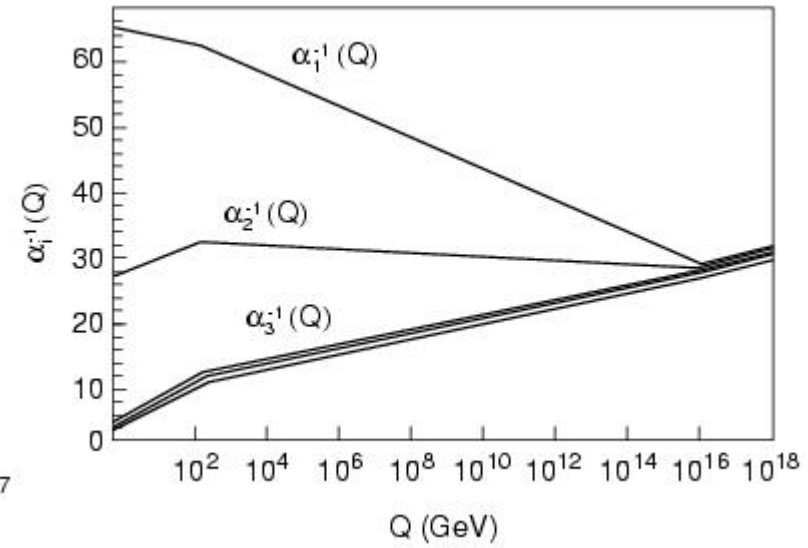
## Other motivations:

- ❑ Generators of supersymmetry, translations and Lorentz-transformations satisfy a common algebra. Consistency suggests that SUSY is local. Local SUSY=theory of gravity (supergravity).
- ❑ Important ingredient in superstring theories.
- ❑ Explains the Higgs mechanism and predicts a heavy top (radiative electroweak symmetry breaking).
- ❑ Gauge coupling unification.
- ❑ Candidates for dark matter.

## Standard Model

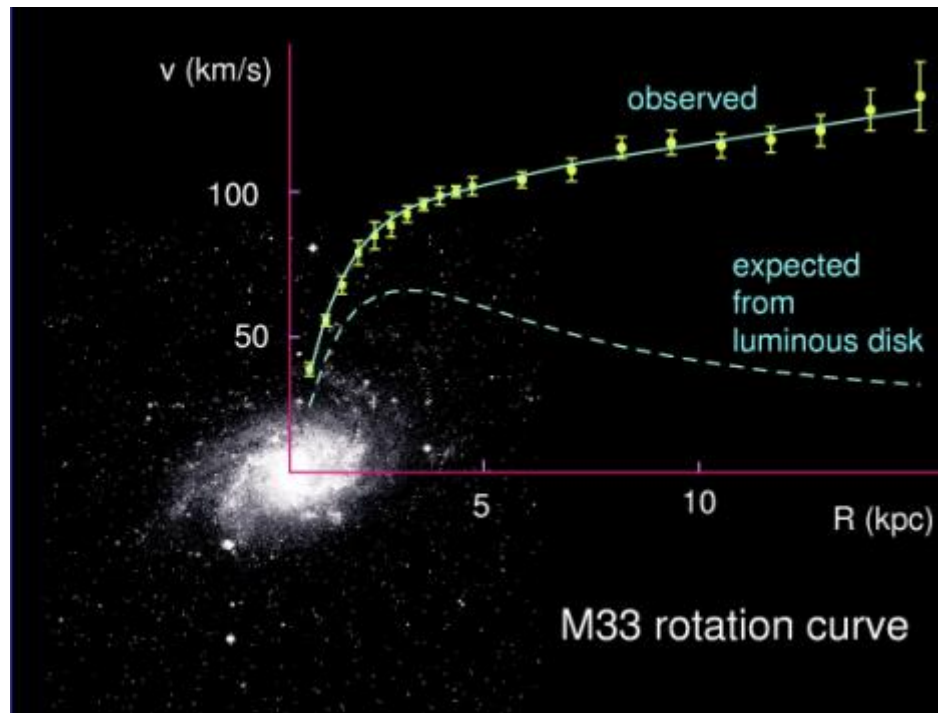


## SUSY



Amaldi, de Boer, Furstenau, Phys. Lett. B 260 (1991) 447

Galaxy rotation curves indicate that galaxies contain unknown matter, which does not emit light, **Dark Matter**.



Dark Matter candidates in SUSY:

neutralinos,  
gravitinos,  
sneutrinos,  
axinos,...

L. Bergström, Rep.Prog.Phys. 2000